PaC-trees: Supporting Parallel and Compressed Purely-Functional Collections

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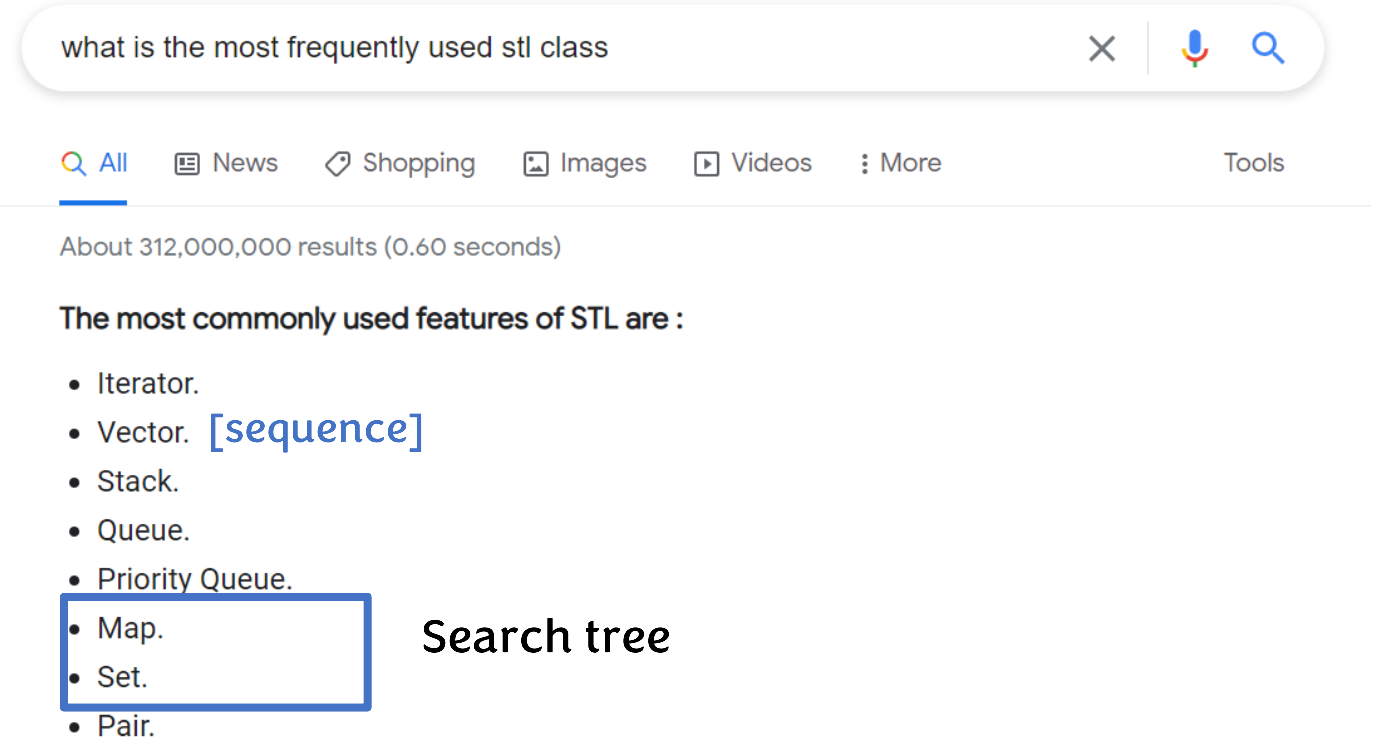
Artifacts available and reusable!
Library available on GitHub:
https://github.com/ParAlg/CPAM

Collection Data Types [sequences, sets, maps]

- A collection of data [e.g., sequences, ordered sets, ordered maps]
- Very commonly-used in programming!
- E.g., in C++ STL: vector, (ordered) set, (ordered) map.
  - Similar in other languages



Collection for inverted index

- **Collection of words**, each mapping to a **collection of documents**

Document 1:
The largest blue whale ever recorded had a length from head to tail of 110 feet and 17 inches.

Document 2:
World's largest blue diamond to come to auction has sold for $57.5 million.

Document 3:
Banging your head against a wall for one hour burns 150 calories.

Word	Document list
...	...
blue	1, 2
whale	1
largest	1, 2
calories	3
diamond	2
head	1, 3
million	2

Collection for inverted index

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Document 1:
The largest blue whale ever recorded had a length from head to tail of 110 feet and 17 inches.

Document 2:
World's largest blue diamond to come to auction has sold for $57.5 million.

Document 3:
Banging your head against a wall for one hour burns 150 calories.

Document 4:
Blue whales eat half a million calories in one mouthful.

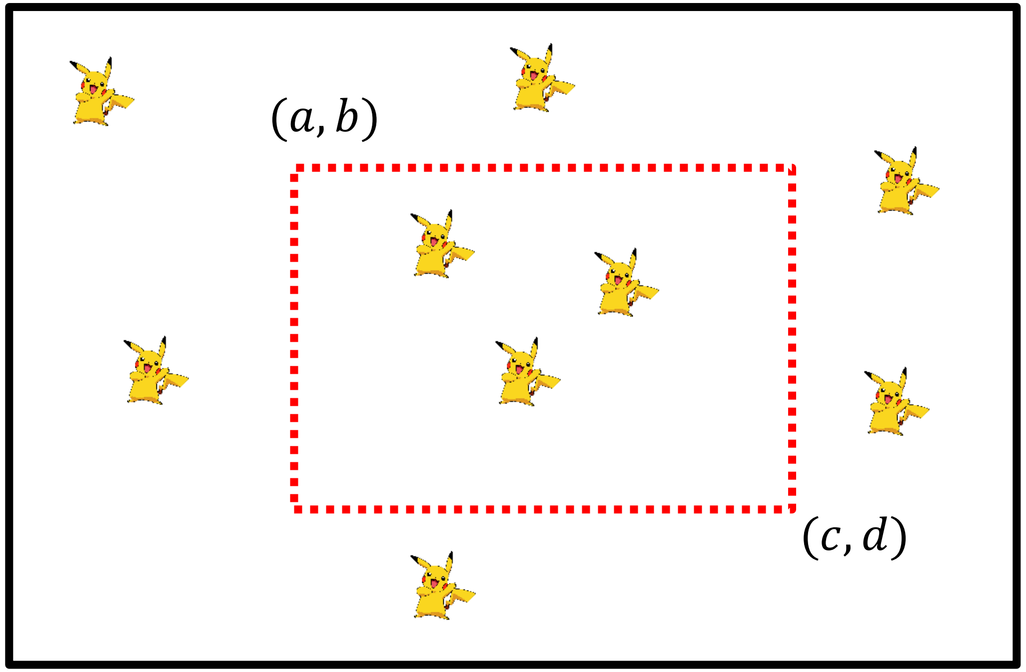
Word	Document list
...	...
blue	1, 2, 4
whale	1, 4
largest	1, 2
calories	3, 4
diamond	2
head	1, 3
million	2, 4
eat	4
mouthful	4

Collection for graph processing

- A collection of vertices, each mapping to a collection of edges

Collection for geometric queries

- A collection of points in 1D or 2D
- Find all points in a certain range



Collection Data Types [sequences, sets, maps] In parallel?

- **[Goal 1] Full interface:** as needed in the applications!

Point updates/queries
find
next/previous
rank/n-th
first/last
insert/delete
……

Bulk updates/queries		
build	flatten	
map	reduce	filter
range	append	reverse
multi-insert/multi-delete		
union/intersection/difference		
……		

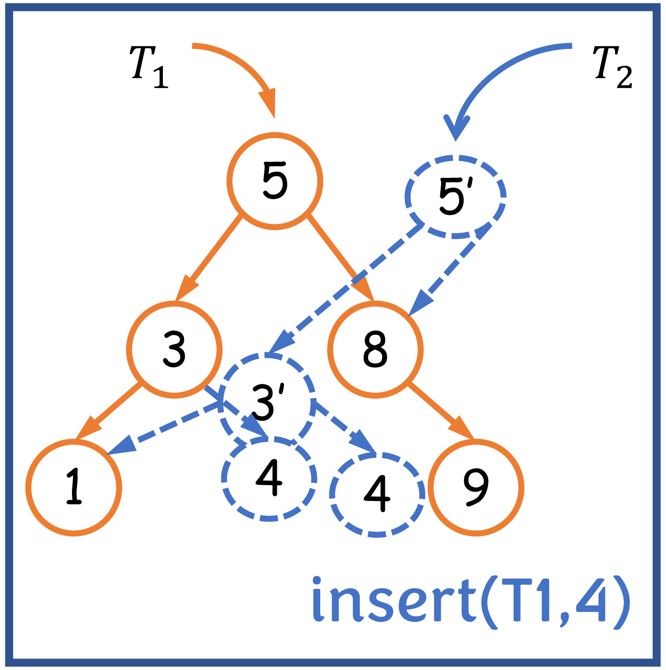
Collection Data Types [sequences, sets, maps] In parallel?

- **[Goal 1] Full interface:** **as needed in the applications!**
- **[Goal 2] Concurrency: Multiple threads can work on the same data structure safely and correctly**
  - **Functional data structure!** [immutable]
  - Each thread works on a snapshot
  - Used in many existing parallel languages/libraries [frie
- **[Goal 3] Parallelism: Bulk operations in parallel**

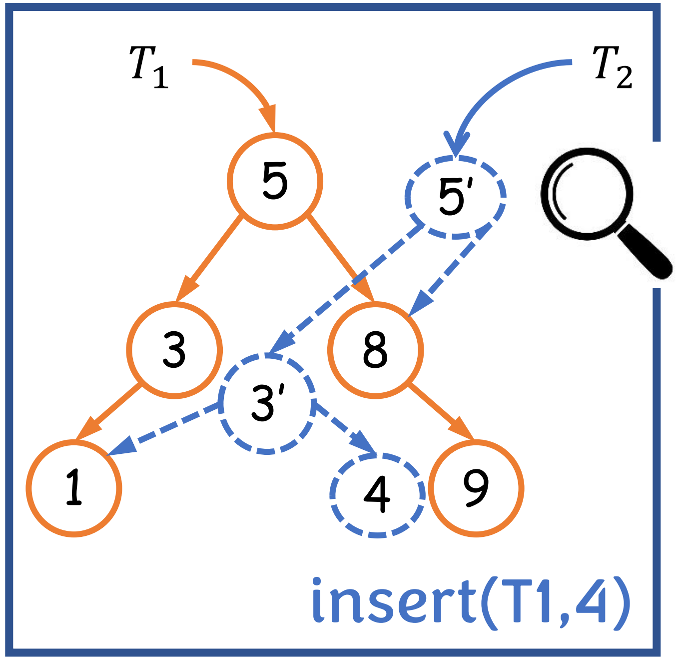
Bulk updates/queries		
build	flatten	
map	reduce	filter
range	append	reverse
multi-insert/multi-delete		
union/intersection/difference		
……		

P-trees [Sun et al., PPoPP'17] for parallel collections

- Parallel binary search trees P-tree in the PAM library
- Functional data structure using path-copying
  - Standard way in functional languages
- General interface for collections: appliable in many applications



P-trees for parallel collections have large space overhead!



Key-value	~8 bytes	
Child pointers	8*2 bytes	24+ bytes
Subtree size	4 bytes	
Ref. cnt.	4 bytes	
Auxiliary info	?	

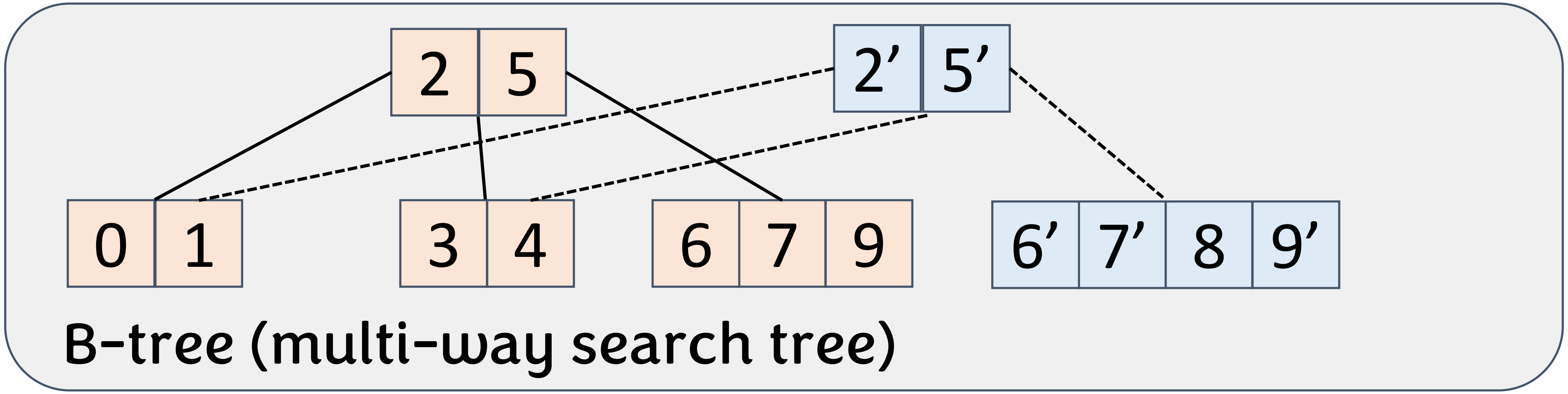
[Goal 4] Space-efficiency: avoid high space overhead!

Our PaC-tree and CPAM library

- full interface of sequences, ordered sets, ordered maps → applicable to a wide range of applications
- functional/immutable
- highly-parallel
- fast both in theory and in practice
- space-efficient!

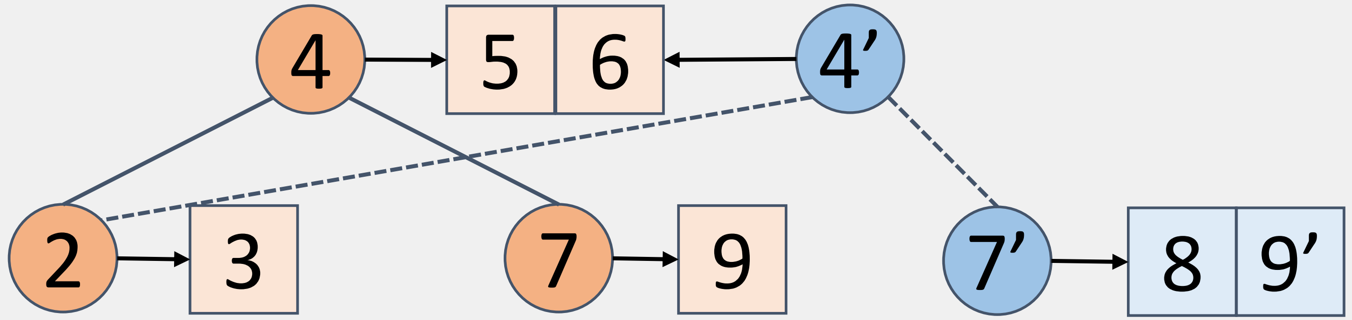
How to Be Space-Efficient?
Put More Data in One Node?

- Multi-way search trees, such as functional B-tree?
- ☹ path-copying is expensive



Put More Data in One Node But Keep the Tree Binary! C-tree in Aspen [Dhulipala et al., PLDI'19]

- Aspen: a graph processing library
- Binary trees with multiple entries in a tree node
- Separate the first entry (called head) for copying
- ☹ Designed for maintaining edges in graphs, not for general collections

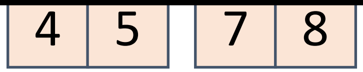


C-tree in Aspen (Compressing nodes in BST)

Keep the Tree Binary
But Put More Data Only in **LEAVES!**
Our new PaC-tree

- **[Balance invariant]** Weight-balanced: left/right subtree sizes differ within a constant factor
- **[Blocking invariant]** Any subtree of size B to 2B will be blocked
- The blocks can be further compressed
- We use **delta encoding**: store the difference relative to the previous

Data:	17	19	24	24	29	33	42	50
	↓	Δ ↓	Δ ↓	Δ ↓	Δ ↓	Δ ↓	Δ ↓	Δ ↓
Encoded data:	17	2	5	0	5	4	9	8



Pac-tree of size 14, B=2

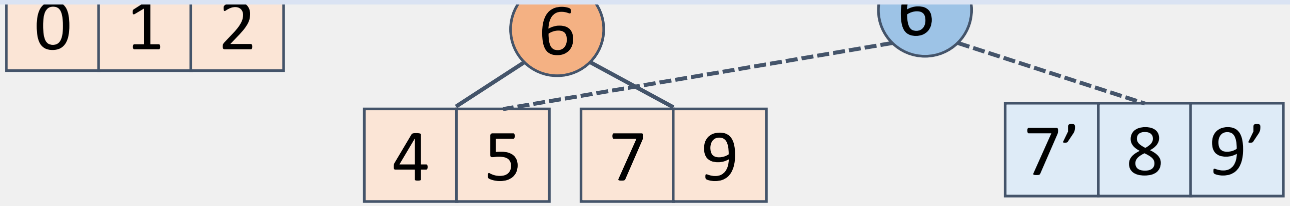
Keep the Tree Binary
But Put More Data Only in LEAVES!
Our new PaC-tree

- [Balance invariant] Weight-balanced: left/right subtree sizes differ within a constant factor
- [Blocking invariant] Any subtree of size B to 2B will be blocked

- Internal

PaC-trees:

- Low space usage
- Parallel and efficient algorithms



(Our new) PaC-tree (Compressing leaves in BST)

PaC-tree – Space Bounds

PaC-trees:

✓ Low space usage

? Parallel and efficient algorithms

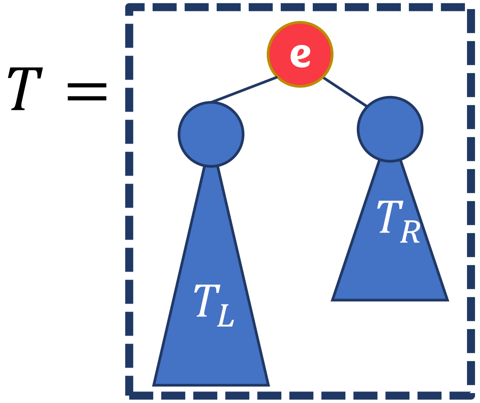
Theorem. The total space of a PaC-tree with block size B + delta encoding, on a set E of n integer keys is:

$$s(E) + O(n/B + B)$$

$s(E)$ = the space to store E in an **array** using delta encoding [lower bound]

Extended Join-based framework in PAM

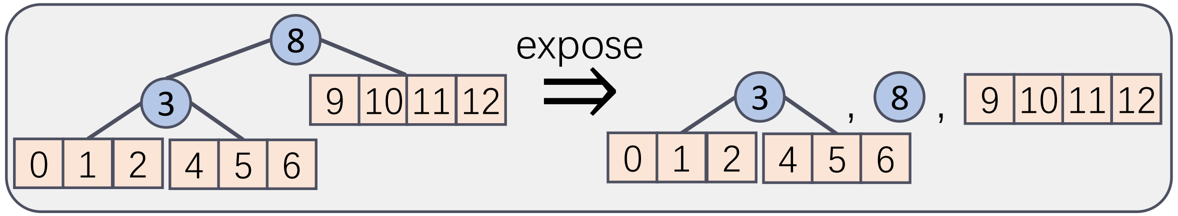
- **The function "join" is a black box – all other algorithms are based on "join"**
- **Path-copying: just copy a few nodes in join**
- $T = \textbf{Join}(T_L, e, T_R)$: T_L and T_R are two trees, e is an entry.
- $T_L < e < T_R$
- **Returns a valid tree** $T = T_L \cup \{e\} \cup T_R$

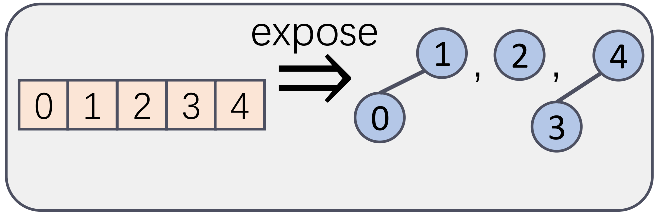


(Rebalance if necessary)

Extended Join-based framework in PAM

- How to extend the algorithms to PaC-trees?
- Deal with the **blocks**?
- Add a primitive expose(T), returns a "left child", a "root" and a "right child"





B=3 in the examples

- We carefully redesigned "join" and "expose" abstractions, and keep the high-level algorithmic ideas in PAM unchanged!
- Keep *blocking invariant* true all the time!

Example: combining two trees

(example for in-place updates. Functional updates can be performed by copying corresponding nodes in the join algorithm.)

union($\boldsymbol{T_1}, \boldsymbol{T_2}$)

if $T_1 = \emptyset$ **then return** T_2
if $T_2 = \emptyset$ **then return** T_1
$(L_2, k_2, R_2) = \text{expose}(T_2)$
$(L_1, b, R_1) = \text{split}(T_1, k_2)$
In parallel:
$T_L = \text{Union}(L_1, L_2)$
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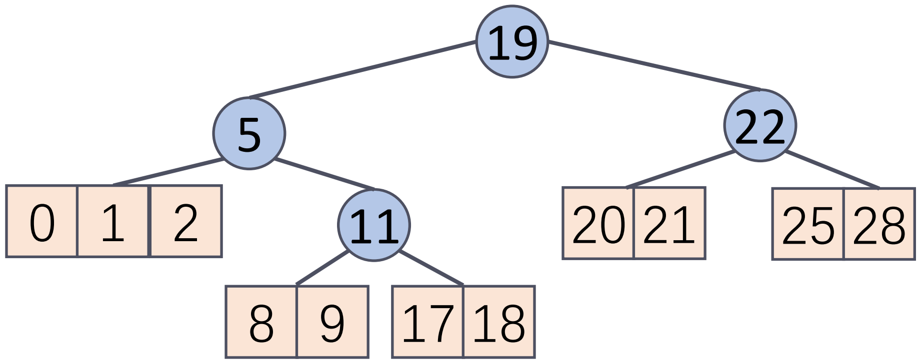

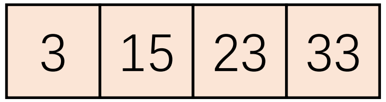

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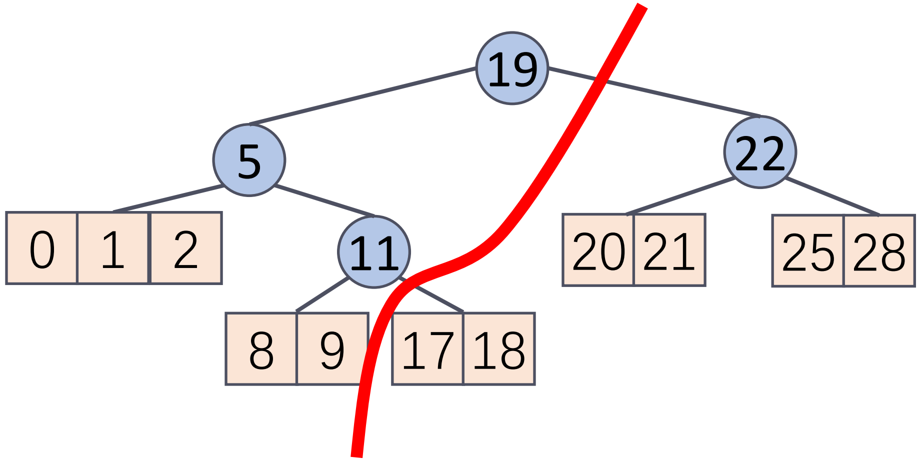

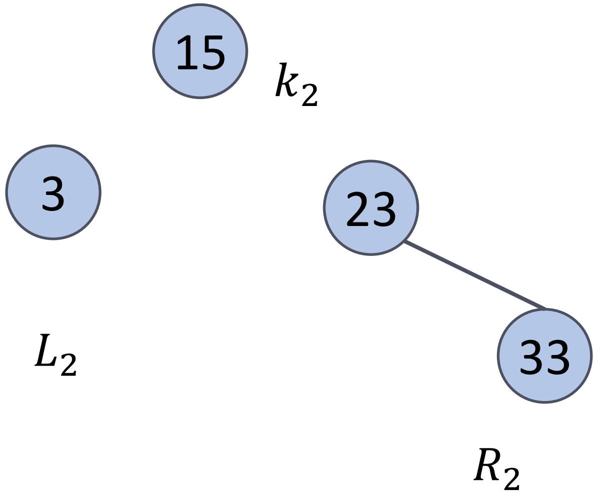

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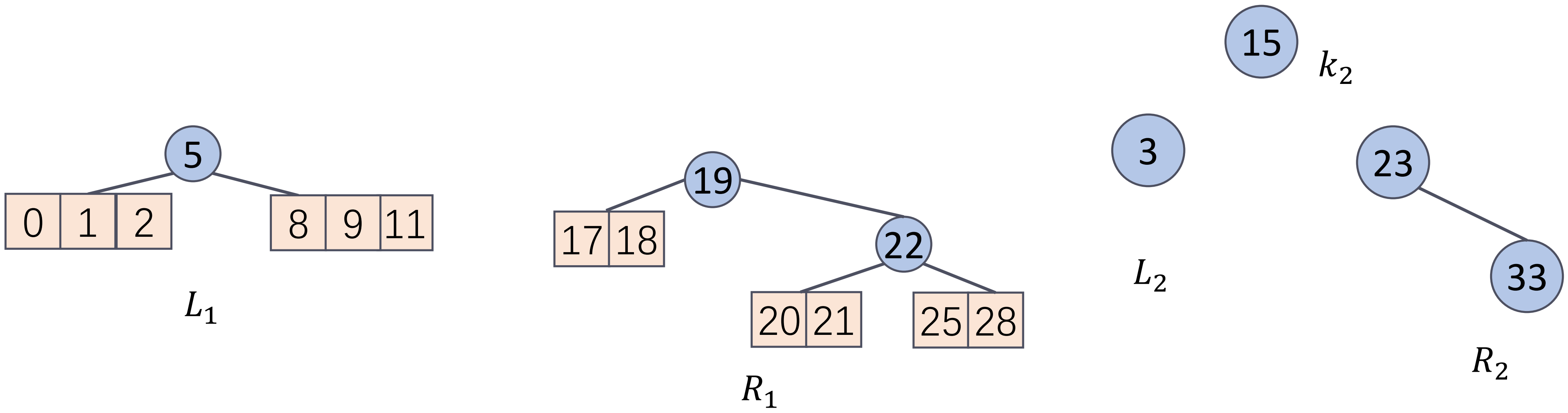

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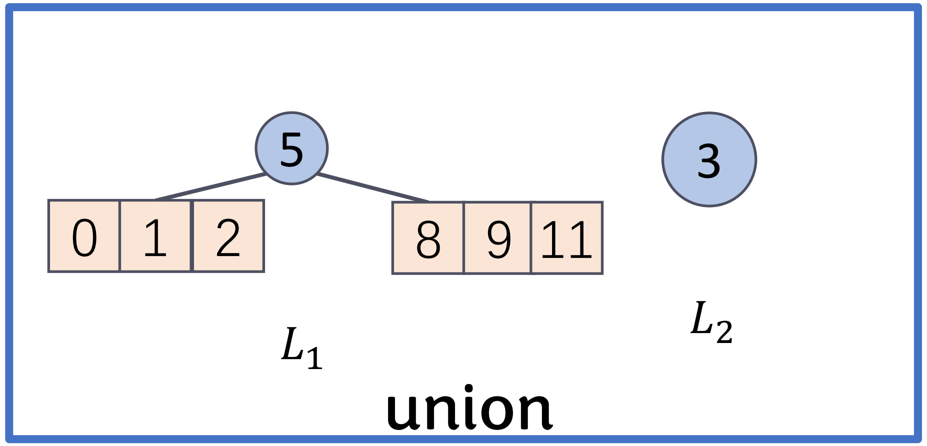

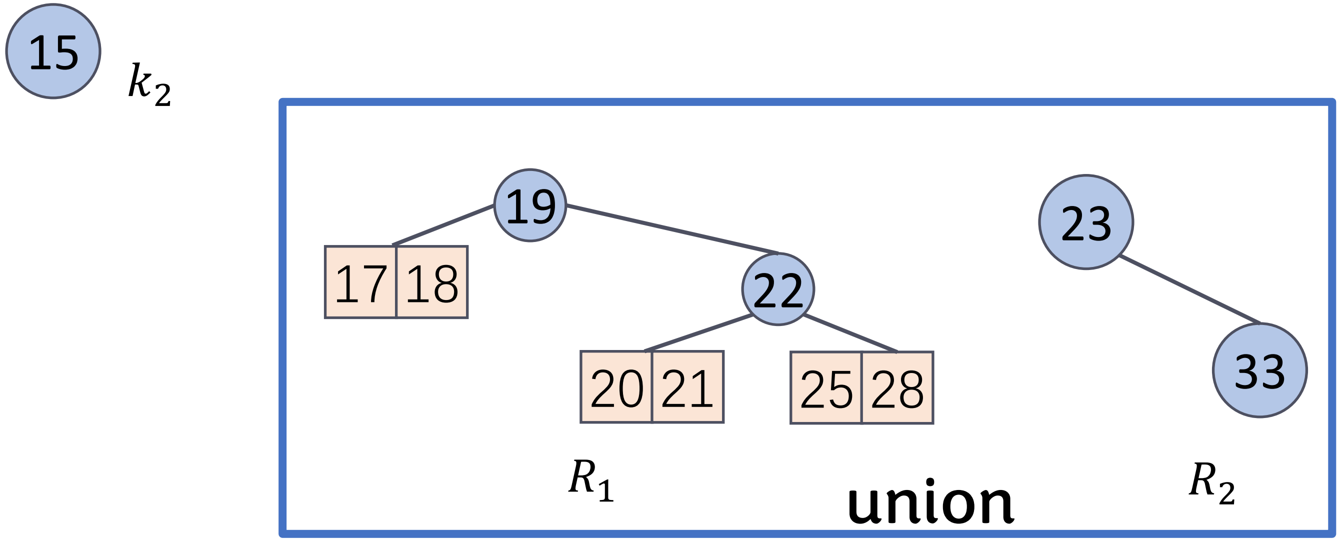

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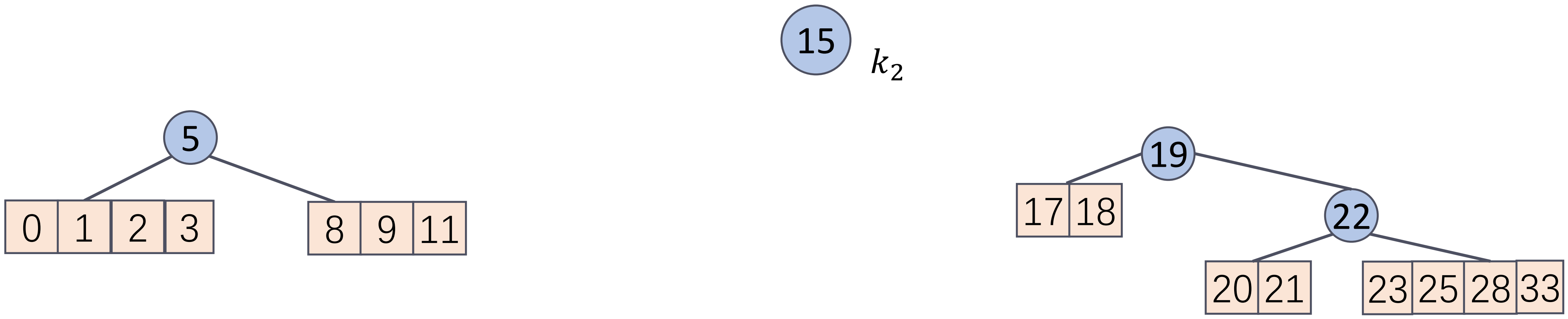

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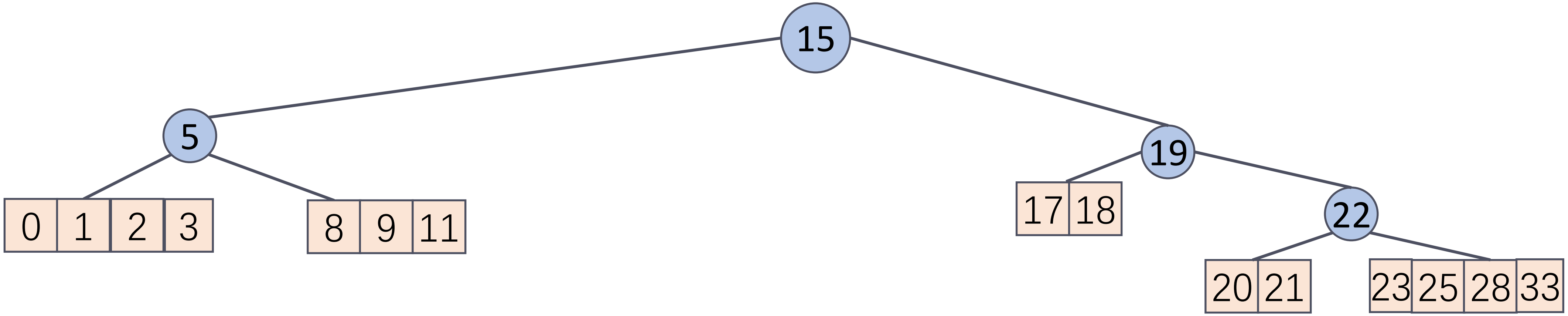


(Theoretical guarantees are provided in the paper)

Lots of Functions and Applications Supported

- **Functions supported**
  - Sequences: Build, map, filter, reduce, take, n-th, findFirst, append, reverse
  - Ordered set and map: (most functions for sequences), next, previous, rank, range, insert, union, intersection, difference, …
  - All of them have theoretical bounds

- **Applications:**
  - 1D interval queries
  - 2D range queries
  - Inverted indexes
  - Graph processing

Experiments

- 72-core Dell PowerEdge R930 (with two-way hyper-threading)
- 1TB of main memory
- Using C++ and the work-stealing scheduler from Parlaylib

Microbenchmarks, compared to P-trees (PAM)

(Functional tree, no blocking leaves or compression)

- PaC-tree (no encoding)

[1.61GB] 2.5x saving

- PaC-tree (encoded)

[0.93GB] 4.3x saving

- P-tree (PAM)

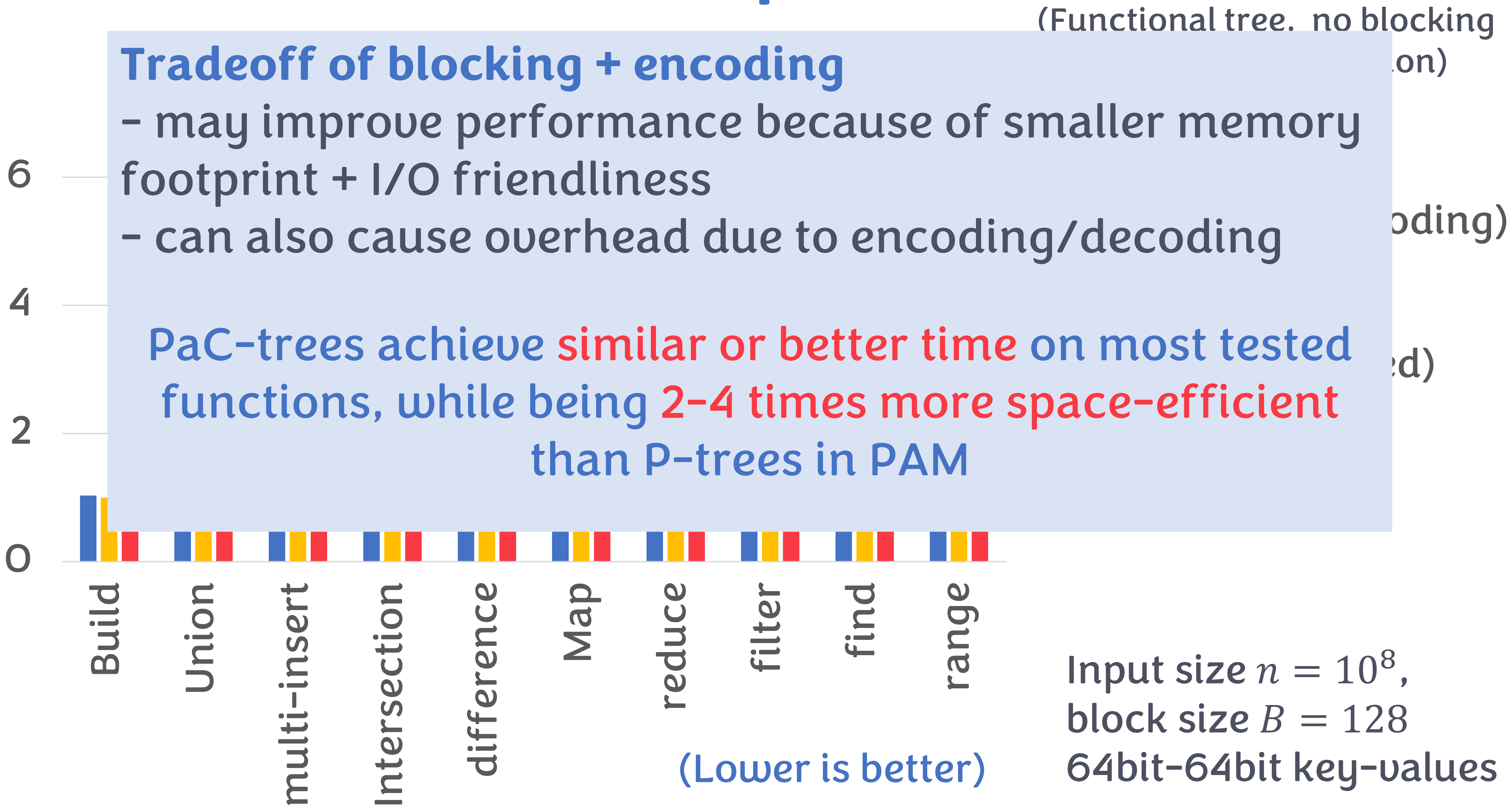
[4.00GB]

Input size $n = 10^8$,
block size $B = 128$
64bit-64bit key-values

Microbenchmarks, compared to P-trees (PAM)

Tradeoff of blocking + encoding
- may improve performance because of smaller memory footprint + I/O friendliness
- can also cause overhead due to encoding/decoding

PaC-trees achieve similar or better time on most tested functions, while being 2-4 times more space-efficient than P-trees in PAM

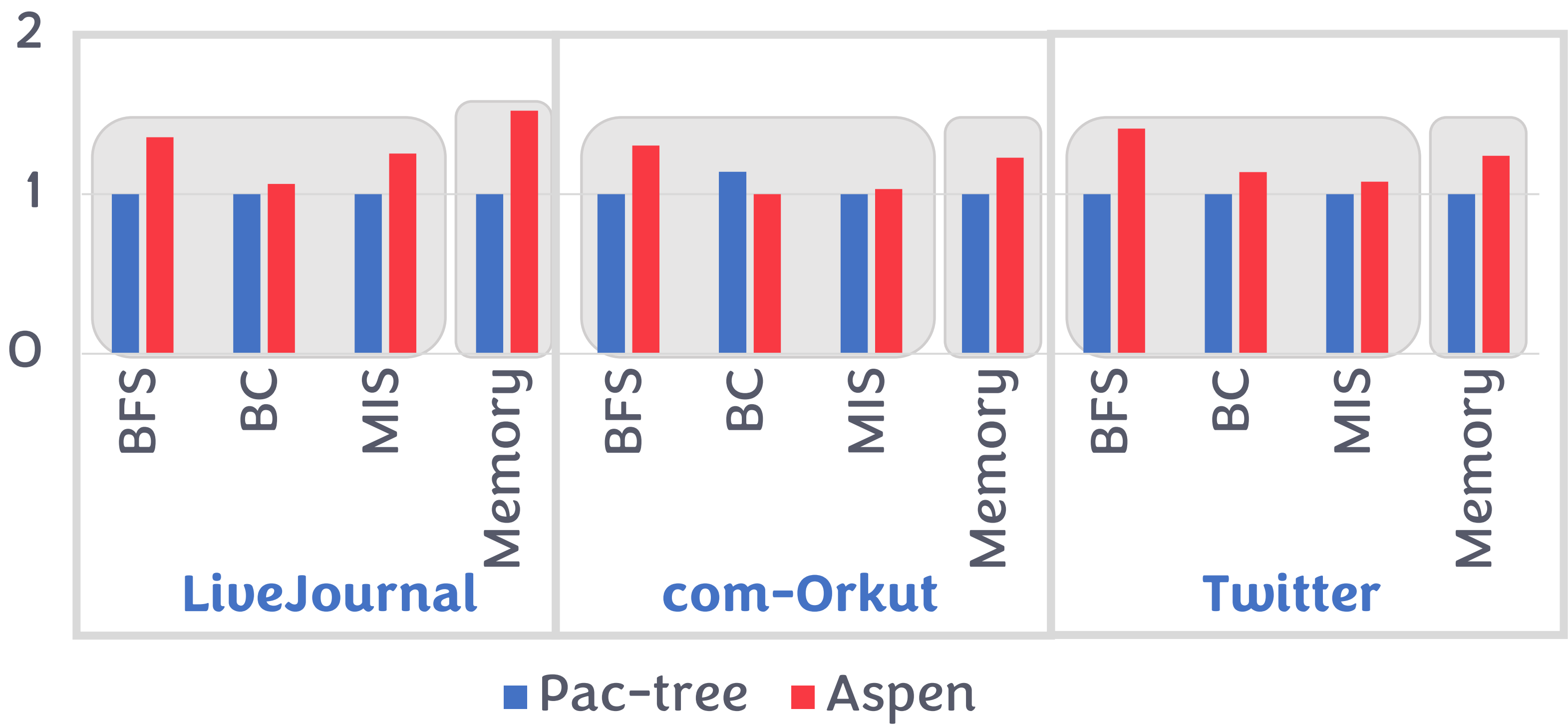


Input size $n = 10^8$,
block size $B = 128$
64bit-64bit key-values

PaC-trees applied to graphs, compared to C-trees (Aspen)

(Functional tree, blocking all tree nodes, specifically for edges in graphs)

Running time/memory relative to the best



Both PaC-tree and Aspen use delta encoding

PaC-tree is almost always faster than Aspen on all benchmarks and graphs

PaC-tree is also 1.2-1.5x more space efficient than Aspen

(Lower is better)

More experiments

- Performance vs. block size
- Space vs. block size
- Inverted indices
- interval tree
- 2D range tree
- graph streaming

- Some of them also requires augmentation, see more details in the paper.

Summary

- **PaC-Tree**
  - Blocked leaves, can be further encoded
  - Provable guarantee in both space and time
  - Safe and efficient for parallelism
- **CPAM library**
  - Full interface for collection for a wide range of applications
  - Outperforms previous non-compressed data structure for collections (P-trees),
    and more space-efficient!
  - Outperforms previous compressed data structure for certain applications (C-trees for graph processing)
    and more space-efficient!

Artifacts available and reusable!
Library available on GitHub:
https://github.com/ParAlg/CPAM