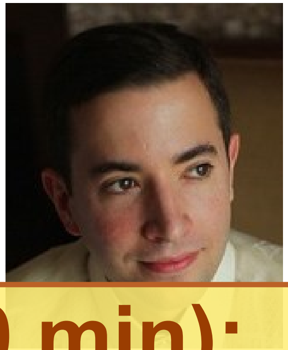
Optimal Parallel Algorithms in the Binary-Forking Model

A joint work with

Yan Gu, UC Riverside

SPAA 2020 (virtual)

July 15, 2020



Guy Blelloch, Jeremy Fineman, Yihan Sun

CMU Georgetown UC Riverside

Full version of this talk (40 min):

https://www.cs.ucr.edu/~ygu/video/bf.mp4

(or just search “Yan Gu UCR”)

1 The binary-forking model

Don't we have lots of computational models already? Why more?

2 Optimal algorithms in the new model

Sorting/semisorting, list/tree contraction, set operations (union, intersect, difference), random permutation, RMQ, etc.

Are they useful and inspiring?

Development of parallel hardware (from 2005)

2020 — **Intel® Xeon® Platinum 8260**

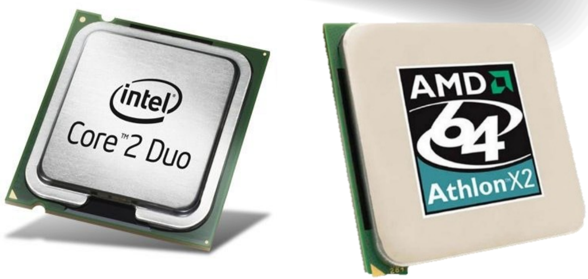
1 of up to 4 sockets:
28 cores
56 threads
38.5 MB LLC
Up to 4.5 TB memory

2017 — **Intel® Xeon Phi™**

72 cores
(288 threads)

For computation intensive applications

2005 — **Intel® Core™2 Duo (E6320) / AMD Athlon 64 X2**



Parallel sorting

[GBSS'15, OKFS'19, AWFS'17, CBB+'15, ABPS'15, BGS'10, BGHS'12, RPC+'11, SBF+'12, MSL+'15 ......]

In-memory databases

AerospikeDBS, Ancelus, Apache Ignite, ArangoDB, Hyper, Kinetica, MemSQL, SQLite, IMDB, SQL Server, Monet DB, SAP HANA, VoltDB, ...

Parallel data structure

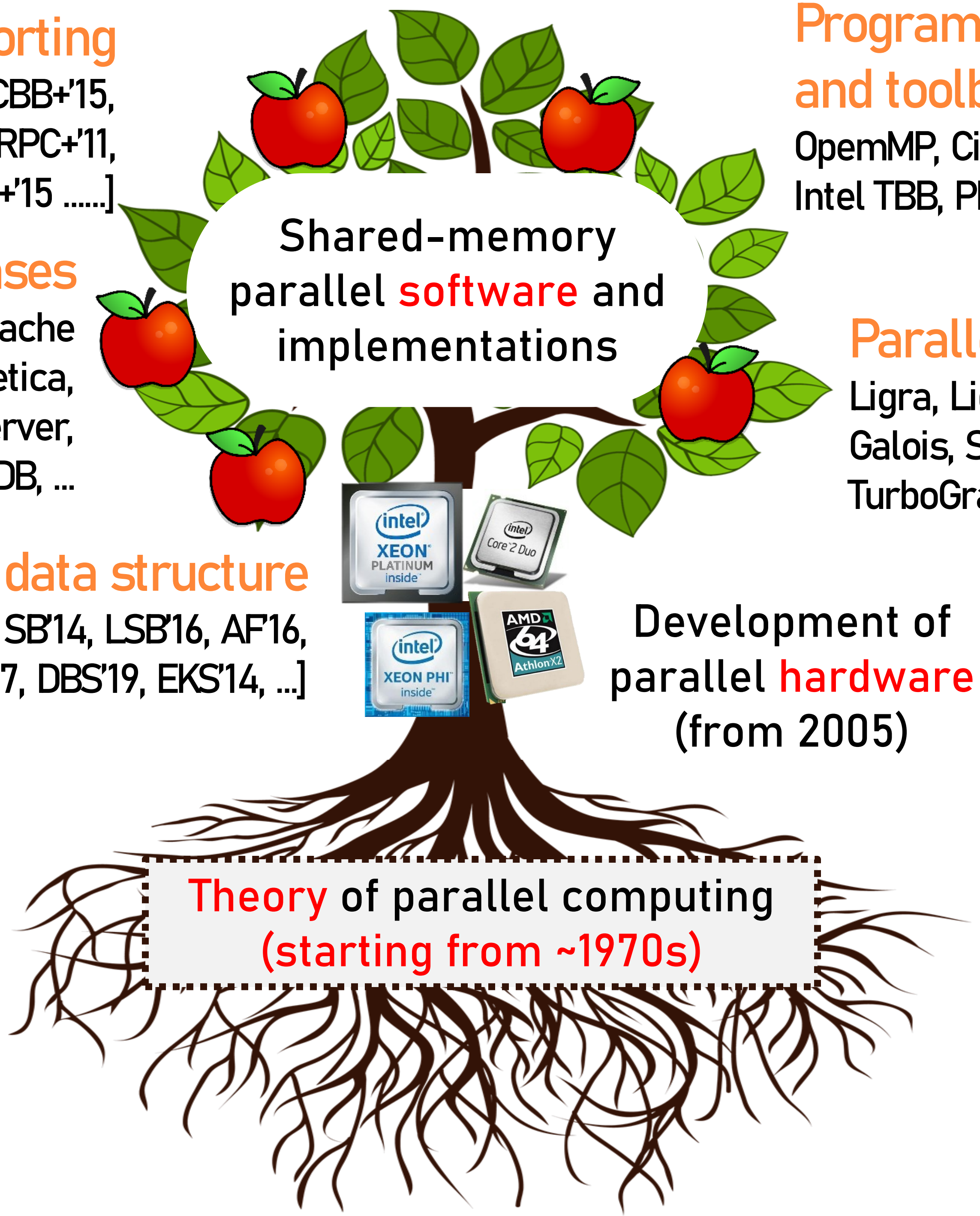
[BFGS'12, BFS'16, SB'14, LSB'16, AF'16, SB'19, YRP'07, BFF+'07, DBS'19, EKS'14, ...]

Programming languages and toolboxes

OpemMP, Cilk, Java fork-join, X10, Intel TBB, PBBS, MCSTL, ...

Parallel graph processing

Ligra, Ligra+, GraphIt, Julienne, Aspen, GBBS, Galois, STINGER, PRISM, X-Stream, Ringo, TurboGraph, FlashGraph, Kaskade, Sage, ...



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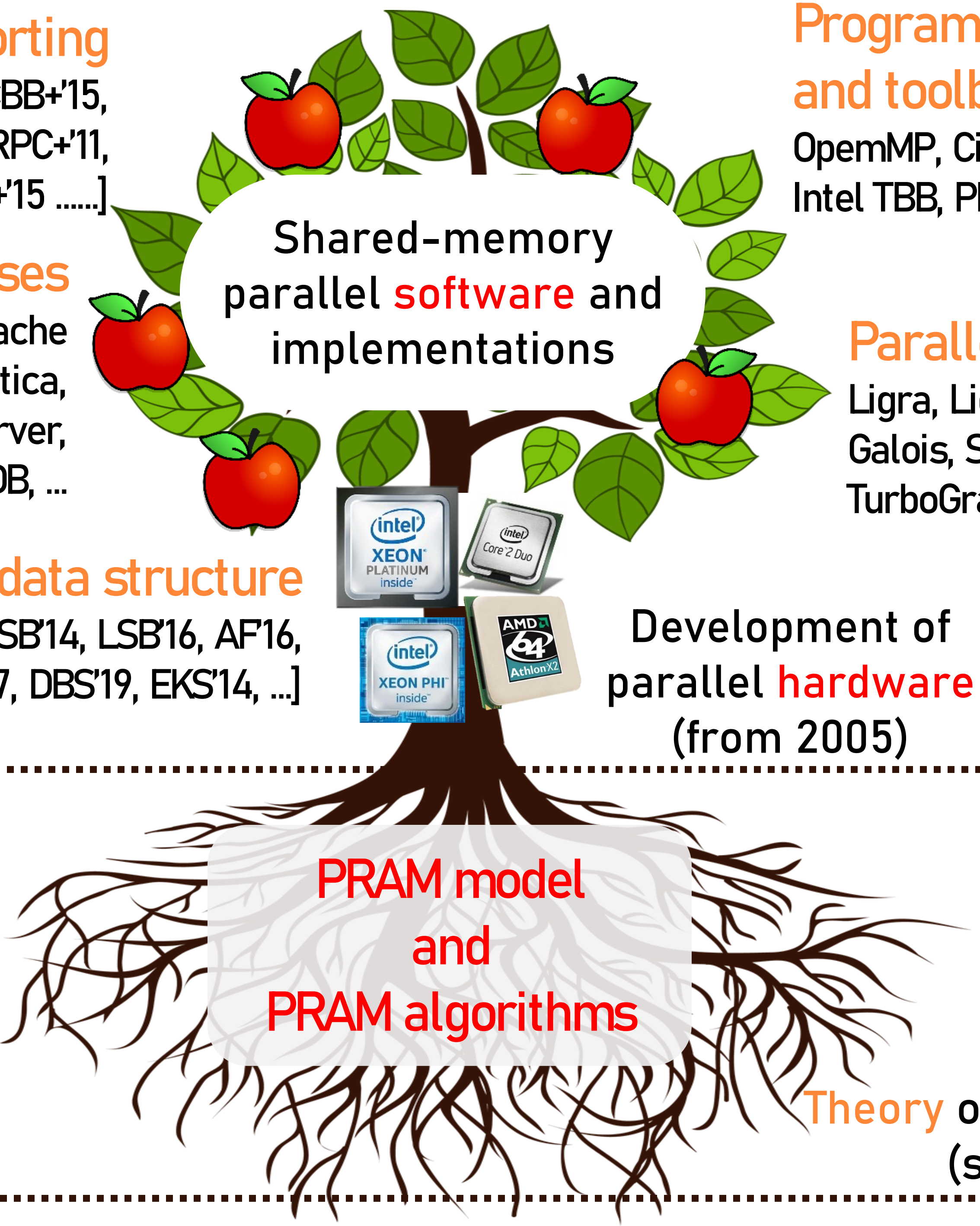
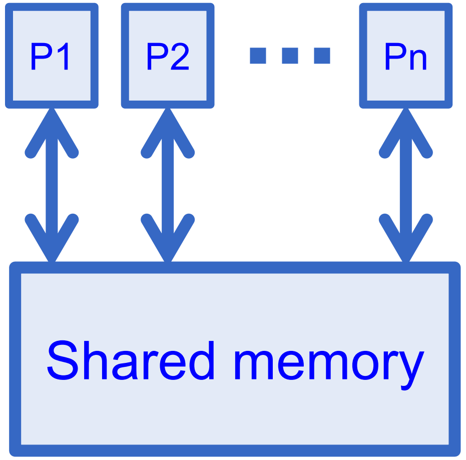
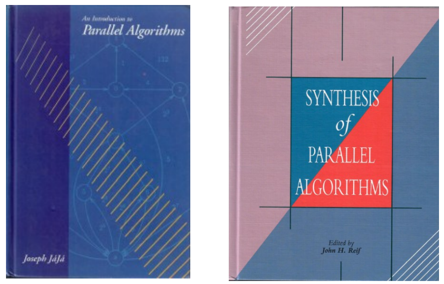


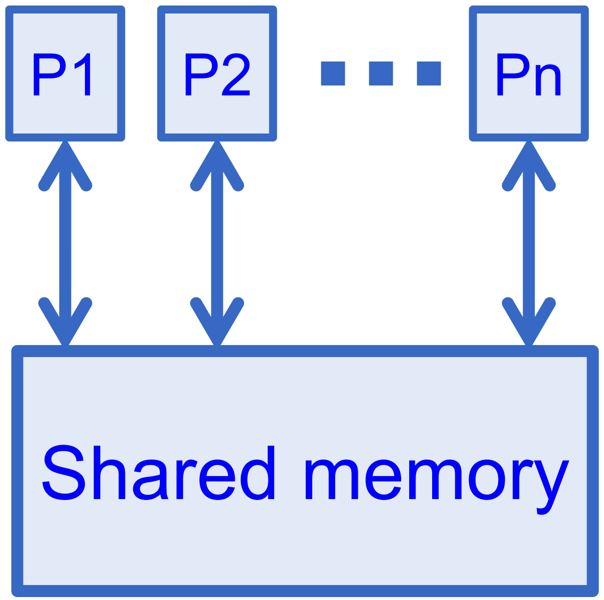
Development of parallel hardware (from 2005)





Theory of parallel computing (starting from ~1970s)

Is PRAM still the best model for designing and analyzing algorithms today?

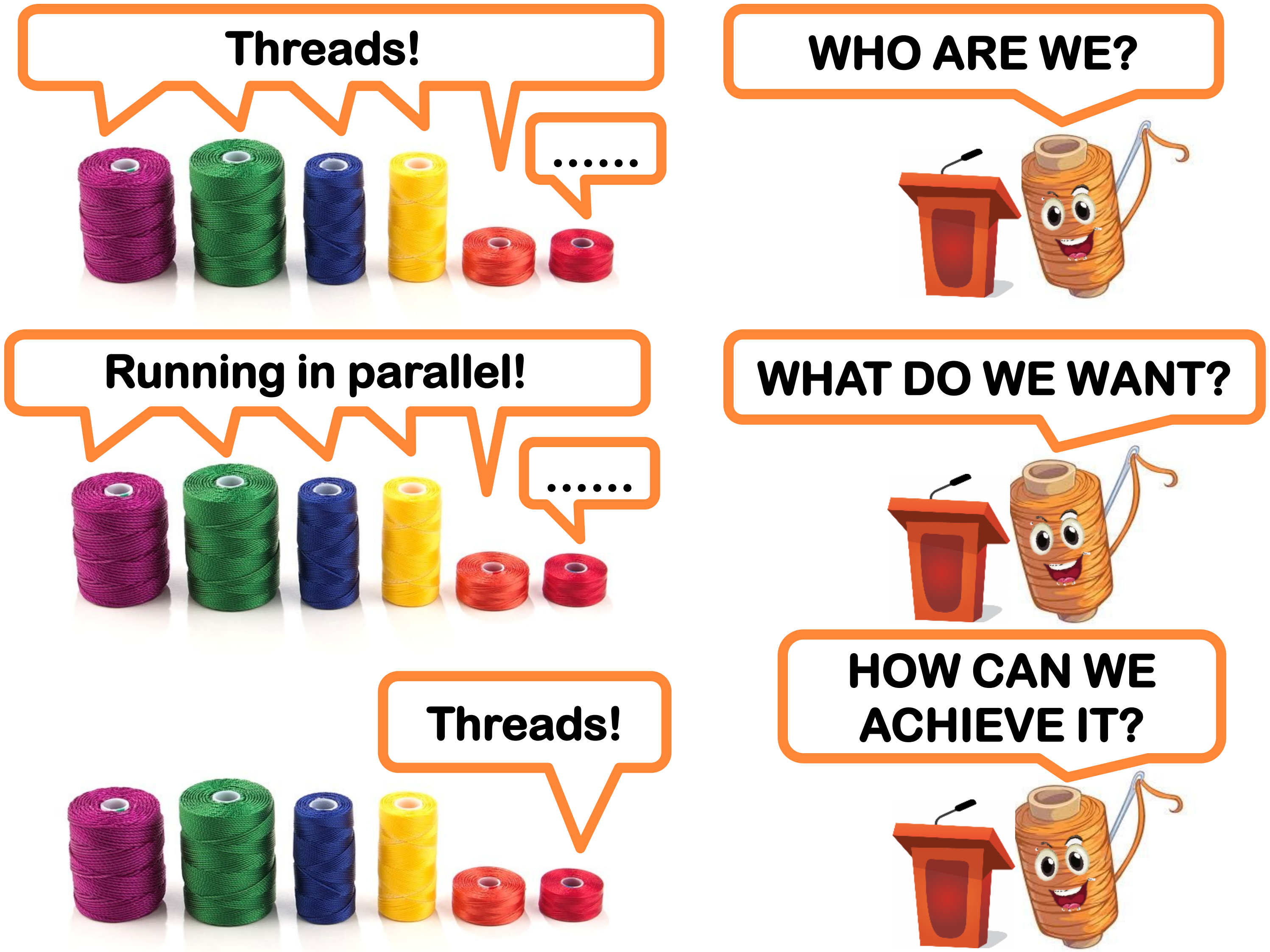


PRAM
(~1980)

Processors run in **lockstep** on a PRAM (ARBITRARY-CRCW, PRIORITY-CRCW, CREW, EREW, CRQW, etc.)

Processors are **highly synchronized**

However, modern architecture is highly asynchronous

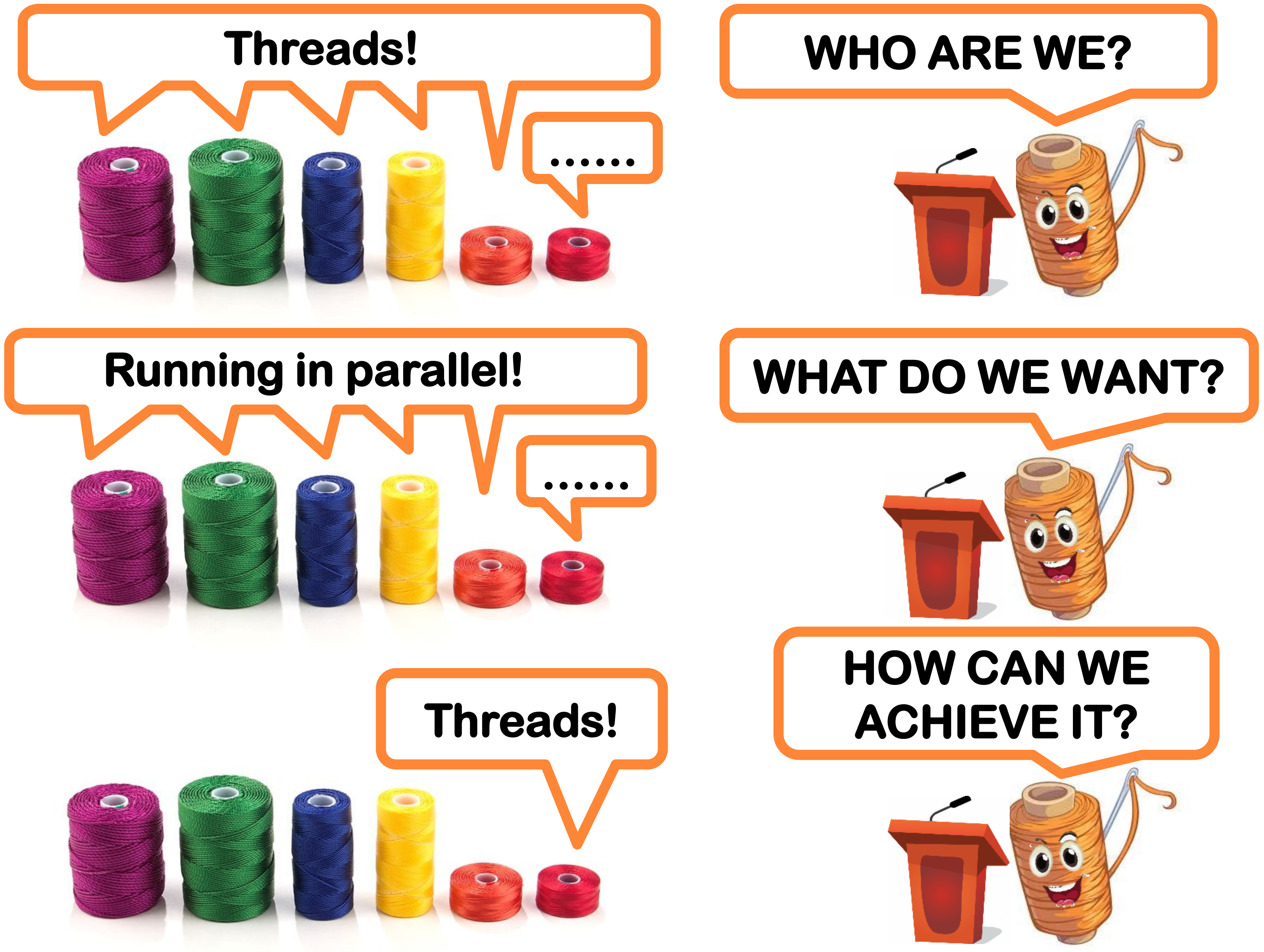


As you can see, threads are highly asynchronous.

Real architecture: processor rates vary significantly due to

- cache effects
- processor pipelines
- ILP
- vectorization
- branch prediction
- hyper-threading
- overclocking
- interrupts
- ……

Global synchronization can be extremely costly



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Real architecture: processor rates vary significantly due to

- cache effects
- processor pipelines
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- vectorization
- branch prediction
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- overclocking
- interrupts
- ……

Global synchronization can be **extremely costly**

- 10^9 arithmetic operations in total
- Adding periodical global synchronization

About $\log n$, **0.7-1.7x** overhead

About $\log^2 n$, **5-13x** overhead

BSP model: give synchronization a higher weight

MPC model: only minimize synchronization rounds

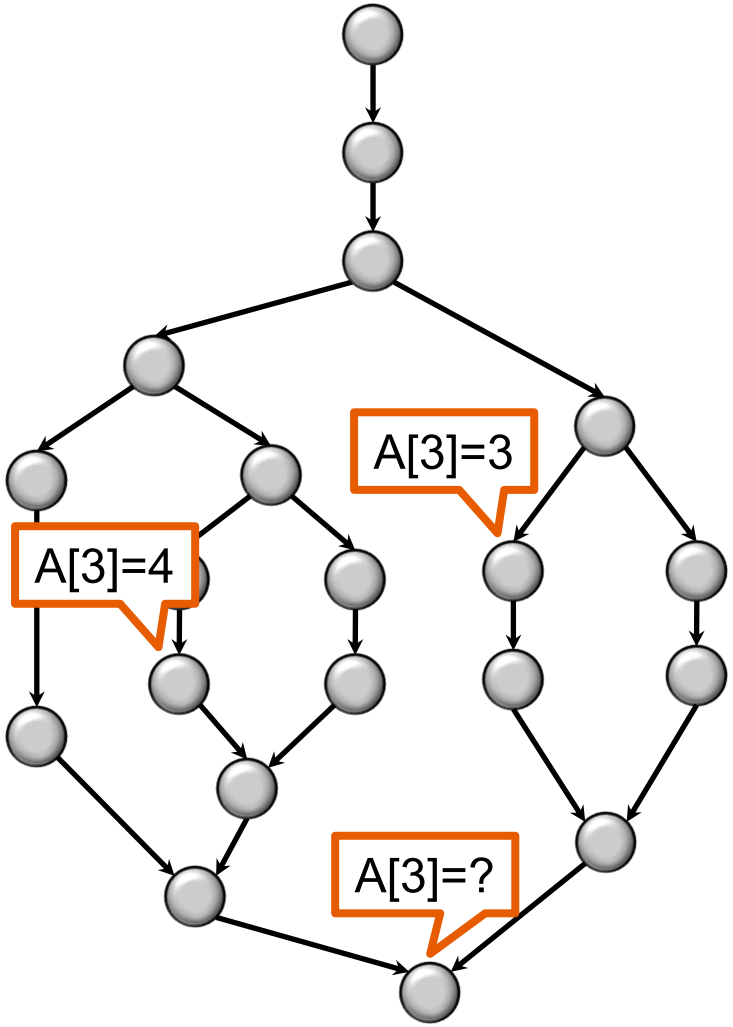
Asynchronized PRAM: synchronize only when needed

The multithreaded models [BL99, ABP01]

- A class of models
- The computational models for many recent parallel algorithms (a short list: [ABB02, AFL+14, BFGS11, BG04, BGS10, BR98, BGSS20, BST12, BL99, CGTT17, CRSB13, CR17, DST16, TYK+15])
  - Yesterday's tutorial #1 (OpenCilk): programming language and interface, runtime supports [BL99, HLL10, SKL+15, SML19]
  - Yesterday's tutorial #2 (parallel trees): efficient tree algorithms both theoretically and practically [BFS16, SB19, SBLP19, SFB18]
- The parallel model in [CLRS, 3rd edition]

The multithreaded model in CLRS

- Similar to a RAM, working on a shared-memory
- A `fork` instruction creates **two** subtasks that can be run in parallel
- After they finish, they `join` and continue
- Assumes **race-free**
  - Or allows some atomic operations such as `Test&Set`, `Compare&Swap`, and `Fetch&Add`



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- A `fork` instruction creates **two** subtasks that can be run in parallel
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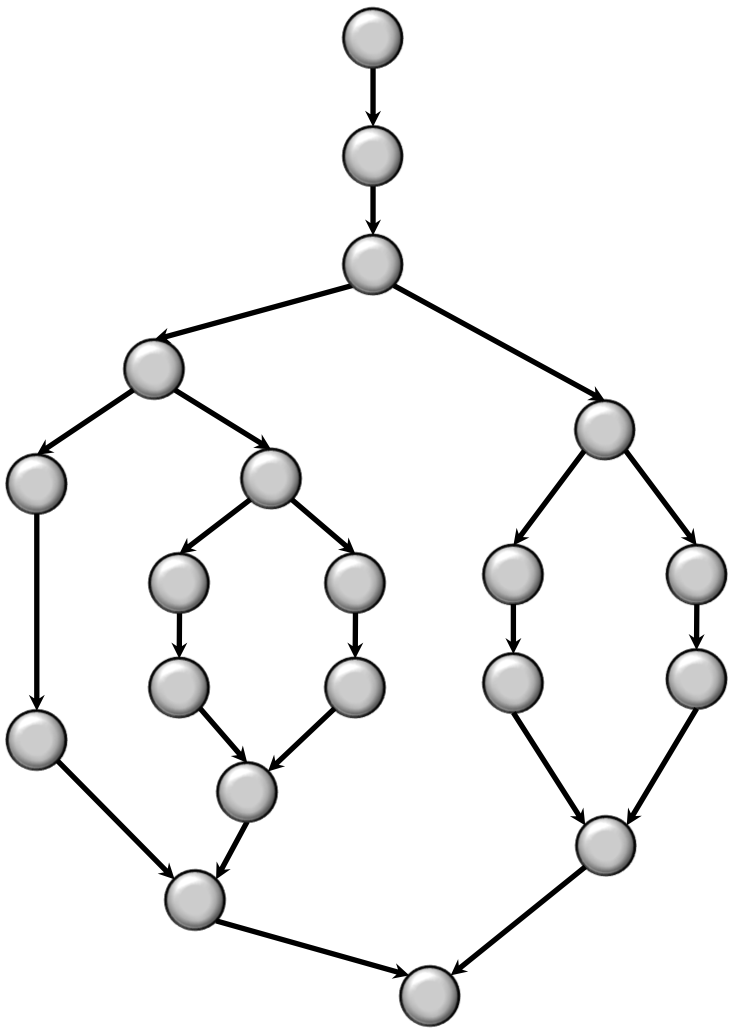
- Assumes **race-free**
- An algorithm is measured by work (#operations) W and span/depth (longest dependence) D

Binary Fork-Join Model

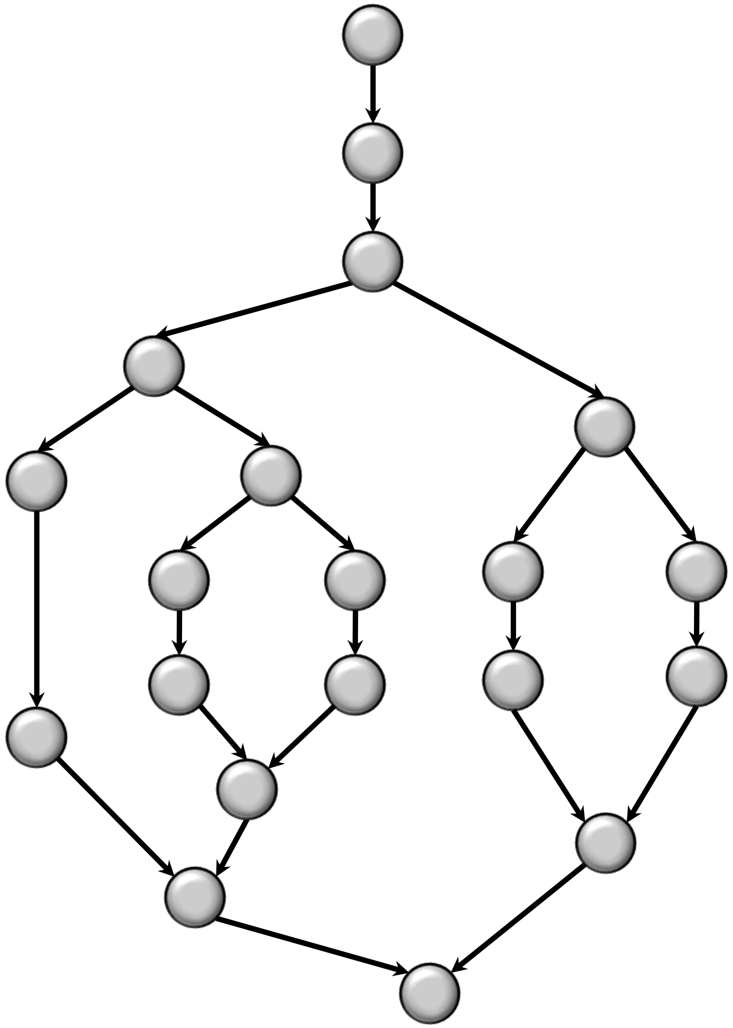


Why multithreaded models?

- Supported by existing libraries such as Cilk, OpenMP, TBB, Java Fork-Join, X10, TPL, Habanero
- Have good theoretical guarantee when mapping to hardware
  - Incur $O(PD)$ "steals" whp using a randomized work-stealing scheduler (under mild assumptions)

#processors   Span/depth

The computation is highly **asynchronous**



Why **revisit** multithreaded models?

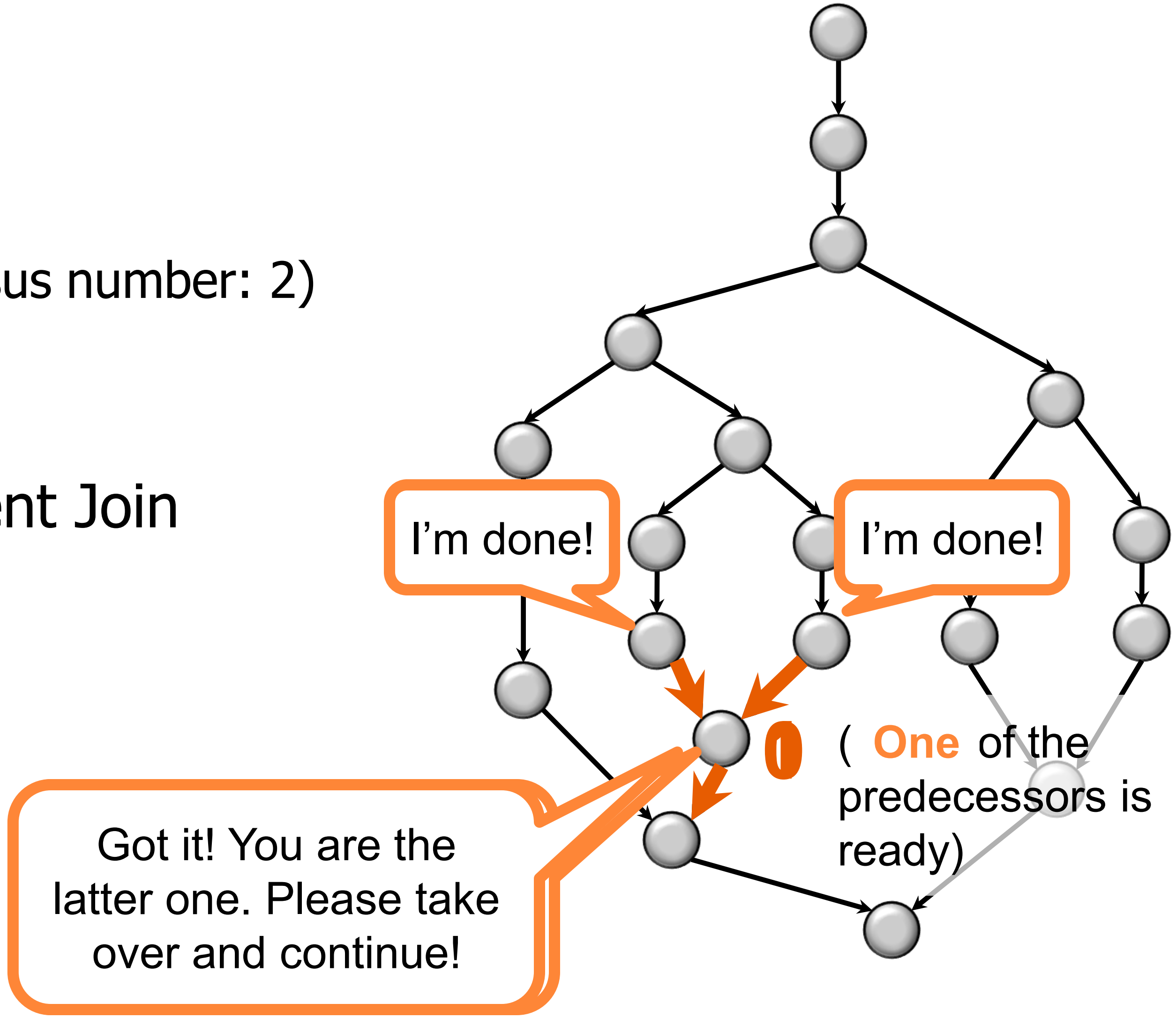
- Decision 1: binary forking vs. n-ary forking
- Decision 2: nested join vs. non-nested join
- Decision 3: race-free vs. assuming `Test&Set` vs. assuming `Compare&Swap` vs. assuming `Fetch&Add`

Rest of this talk

1. The binary-forking model
2. Algorithm design inspired in the asynchronous setting

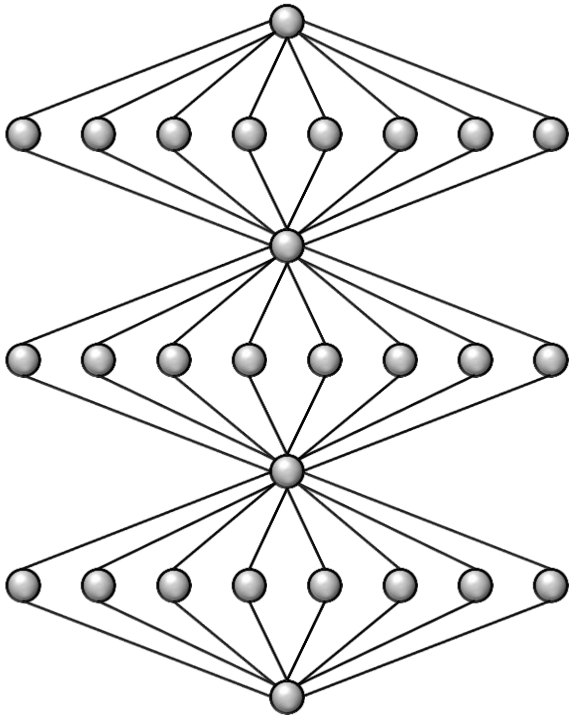
The binary-forking (BF) model

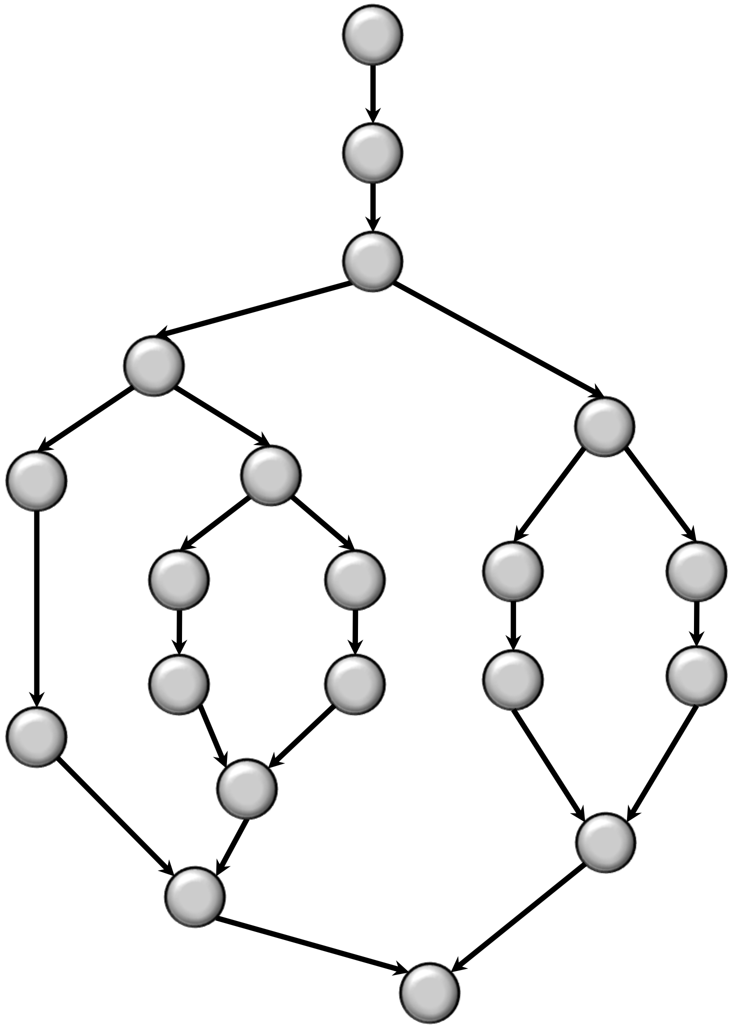
- `Fork` generates two subtasks
- Assumes `Test&Set`
  - Atomically set a bit flag from 0 to 1
  - Weakest consensus primitive (consensus number: 2)
- `Test&Set` can be used to implement Join



Q1: why **binary** when forking?

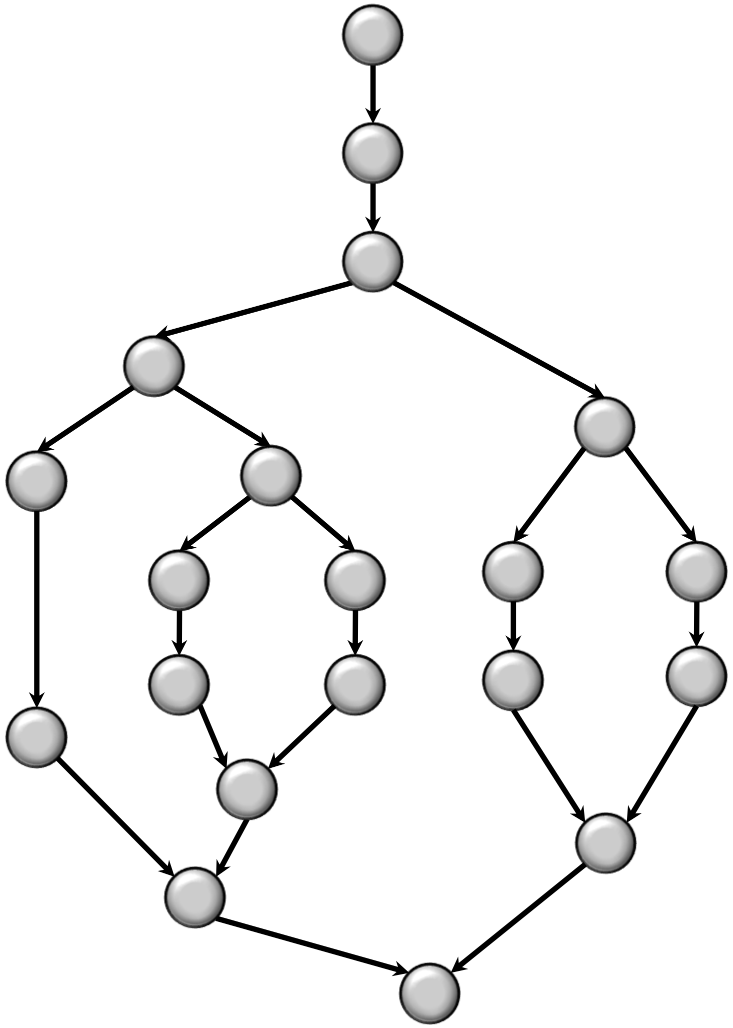
- In practice, that's how most existing software is implemented
- Arbitrary forking = global synchronization





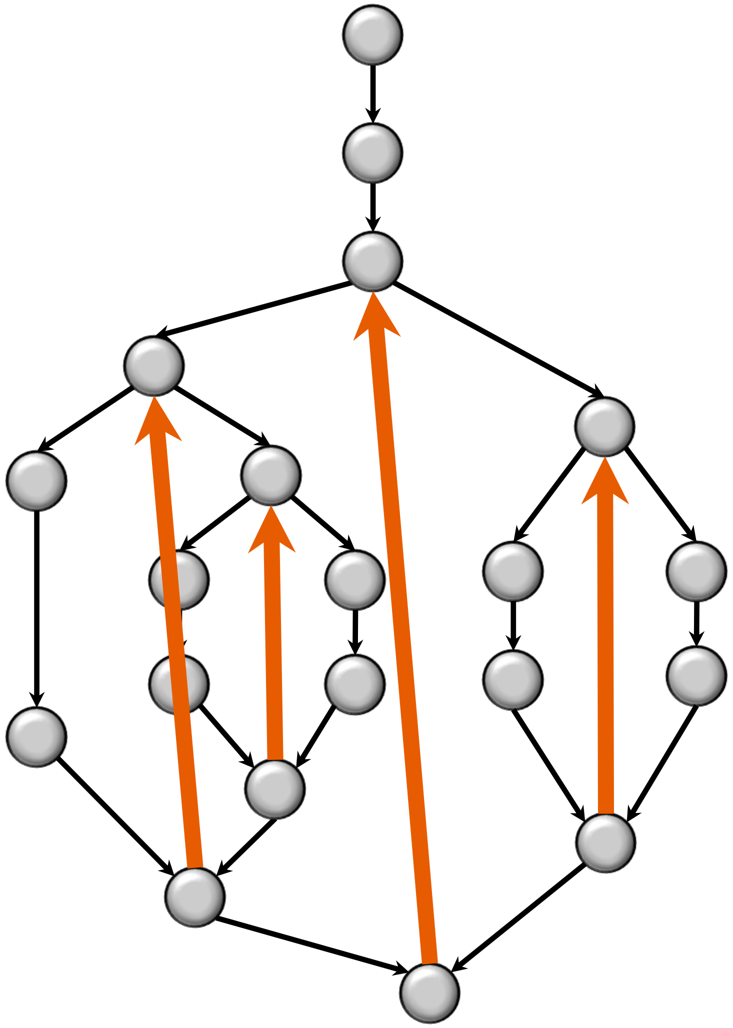
Q1: why **binary** when forking?

- In practice, that's how most existing software is implemented
- Arbitrary forking can abuse global synchronization (no difference as compared to the PRAM model)
- For dynamic work-stealing schedulers, most efficient scheduling results (e.g., **[BL99, ABP01, ABB02]**) hold only for **binary** forking
  - $O(PD)$ "steals" whp ($O\left(\frac{W}{P}+D\right)$ time)
  - Otherwise, the span bound can increase by $O(\log n)$



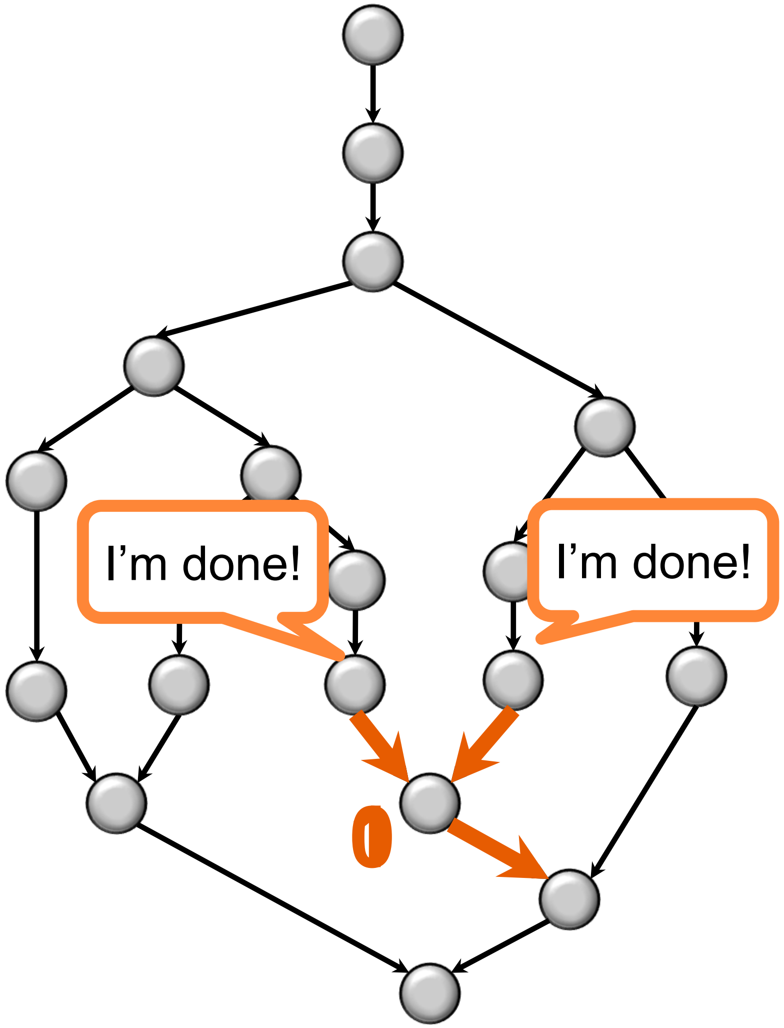
Q2: is **nested** join necessary?

- Easy to implement
  - Cilk: `cilk_spawn A(); B(); cilk_sync;`
  - Lambda: `par_do([&](){A();}, [&](){B();});`



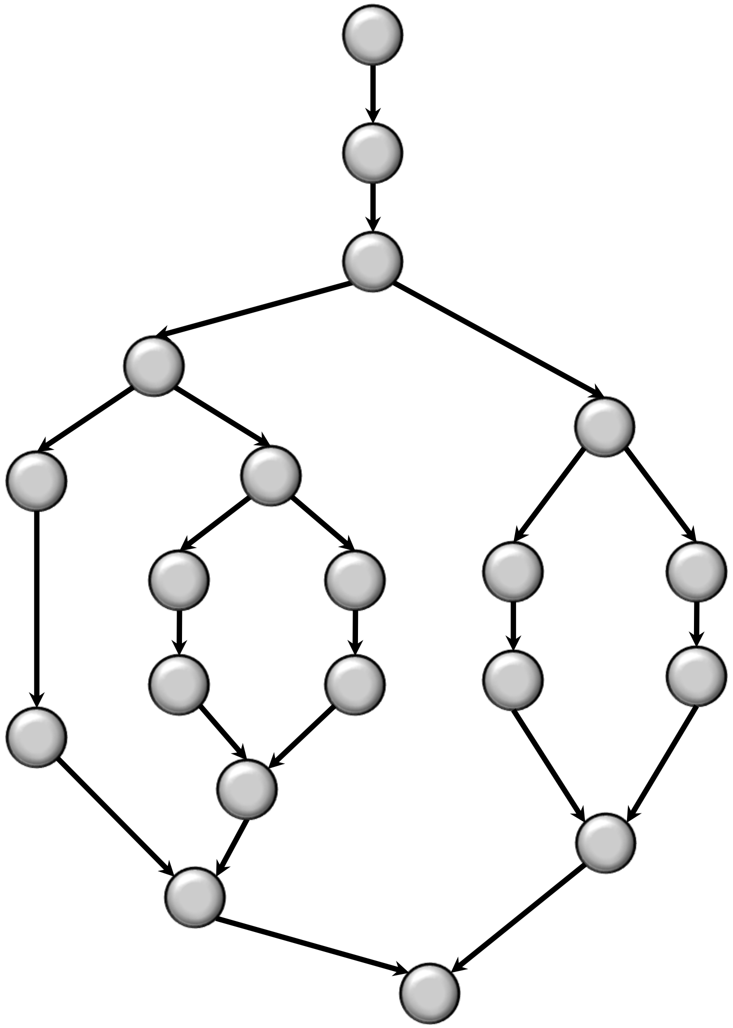
Q2: is **nested** join necessary?

- Easy to implement
  - Cilk: `cilk_spawn A(); B(); cilk_sync;`
  - Lambda: `par_do([&](){A();}, [&](){B();});`
- Scheduling results for work-stealing (e.g., **[ABP01, ABB02]**) do not require **nested** join
- In BF model, `Test&Set` allows non-nested join



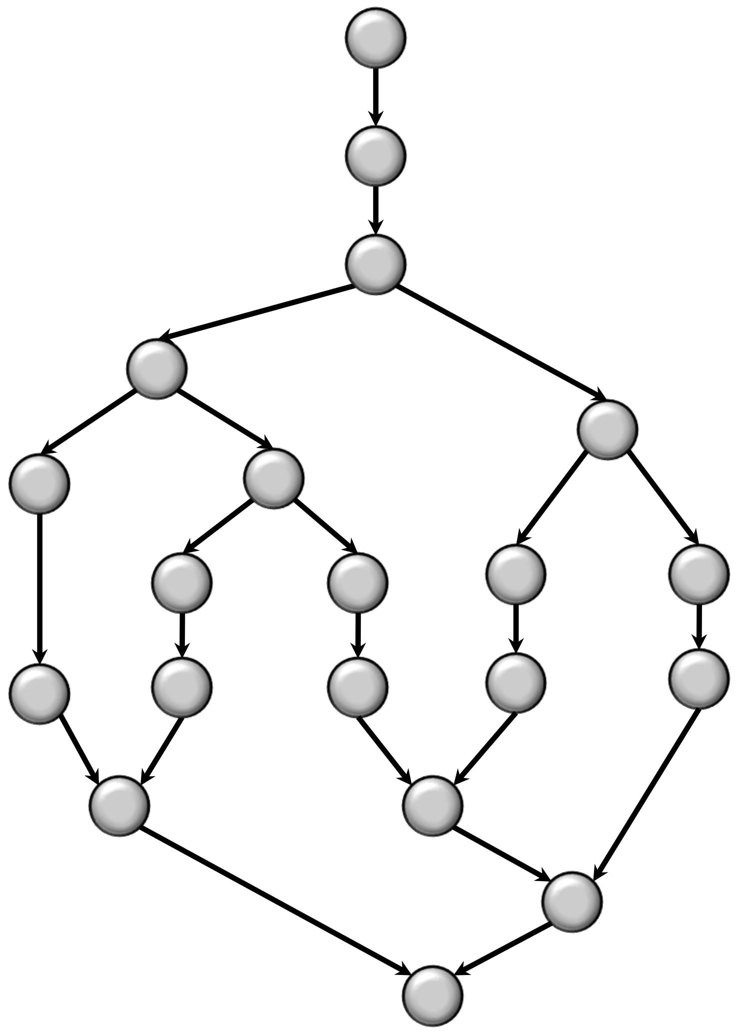
Q3: Why assume `Test&Set`?

- What is `Test&Set`?
  - Atomically set a bit flag from 0 to 1
  - Weakest consensus primitive (consensus number: 2)
- `Join` needs at least `Test&Set`
  - Two threads need to reach consensus
  - No stronger assumptions for real hardware
- Real architecture supports `Test&Set`



The binary-forking (BF) model

- `Fork` is binary
- Assumes `Test&Set`
  - Atomically set a bit flag from 0 to 1
  - Simplest consensus primitive (consensus number: 2)
- `Test&Set` can be used to implement Join (non-nested join)
- Theoretical scheduling guarantees still hold
  - Incur $O(PD)$ “steals” whp using a randomized work-stealing scheduler



The two derived models

- The binary fork-join model **[CLRS, 3rd edition]**
  - Assumes `Join` as a primitive, instead of `Test&Set`
  - Additional guarantees for race-free and determinism
- The binary-forking model with `CAS`
  - Assumes `Compare&Swap`, a stronger consensus primitive (consensus number: ∞)
  - Easier for practical programming

Outline of this talk

1. The binary-forking model

2. Algorithm design inspired in the asynchronous setting

Some PRAM algorithms are already optimal, but not all

- Lower bound: $\Omega(\log n)$ span for algorithm with $\Omega(n)$ work
- A parallel-for of size n has $O(\log n)$ span in the BF model
- Reduce, scan, filter are already optimal
- Cole's ingenious PRAM mergesort requires $O(\log n)$ rounds
  - Span on BF model is $O(\log n) \cdot O(\log n) = O(\log^2 n)$, the same as a normal mergesort
  - Asynchronous samplesort **[BGS10, BAD20]** with $o(\log^2 n)$ span is faster in practice

Optimal parallel algorithms in the BF model

- $O(\log n)$ span ➔ none or a constant number of global synchronization
- New results for all these fundamental problems
- Surprisingly, these algorithms are all quite simple

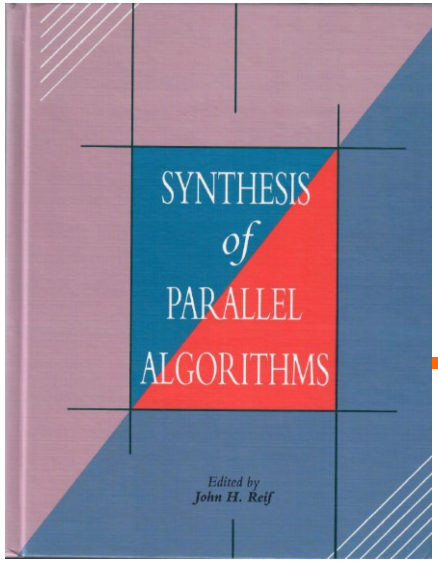
Problem	Work	Span/ depth
List contraction	$O(n)$	$O(\log n)^*$
Comparison sort	$O(n \log n)^\uparrow$	$O(\log n)^*$
Semisort	$O(n)^\uparrow$	$O(\log n)^*$
Random permutation	$O(n)^\uparrow$	$O(\log n)^*$
Range minimum query	$O(n)$	$O(\log n)$
Tree contraction	$O(n)$	$O(\log n)^*$
Ordered-set Operations (Union, intersect, diff)	$O\left(m \log\left(\frac{n}{m}+1\right)\right)$	$O(\log n)$

$^\uparrow$ expected, * with high probability

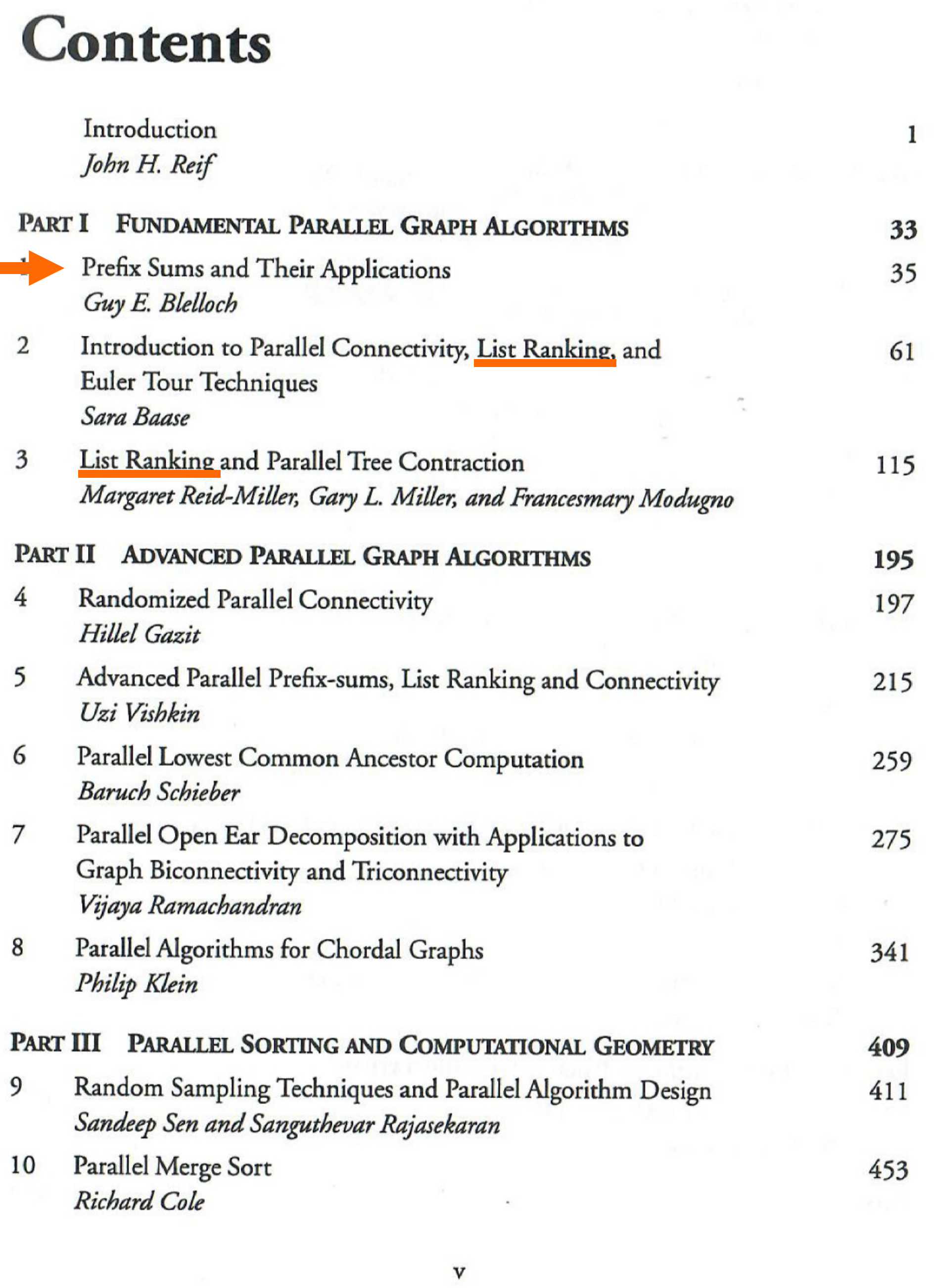
List ranking

- Prefix sum (scan)

[1, 2, 3, 4, 5] → [1, 3, 6,

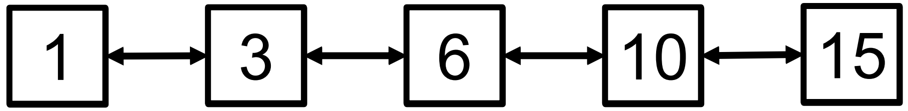


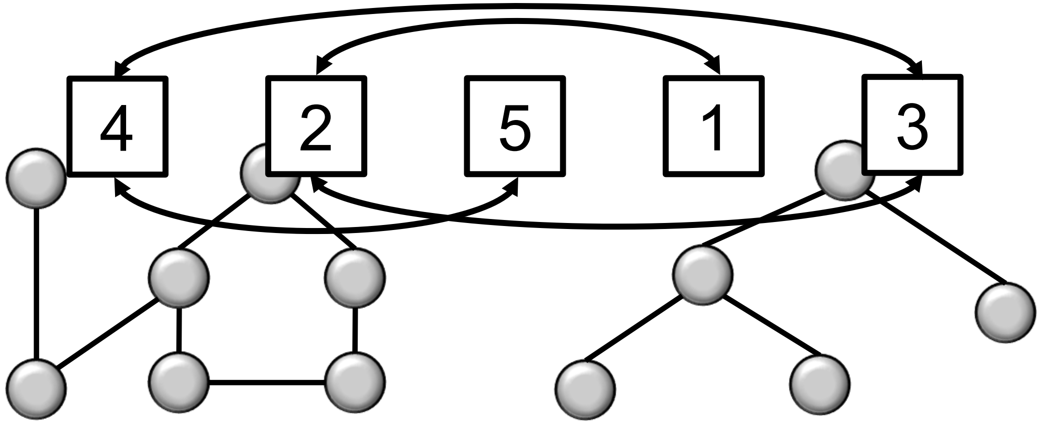
Synthesis of Parallel Algorithms



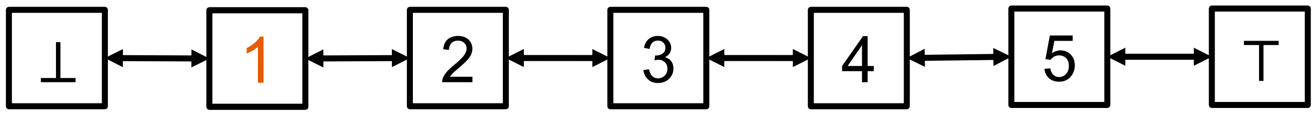
Contents

- List ranking





The splice operation for list ranking



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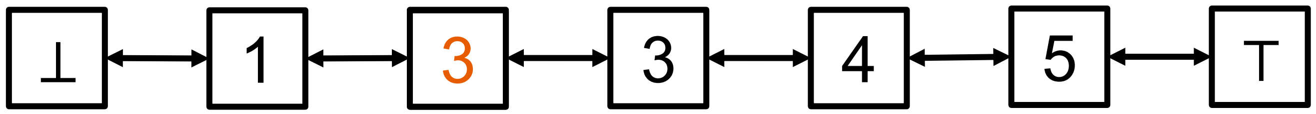

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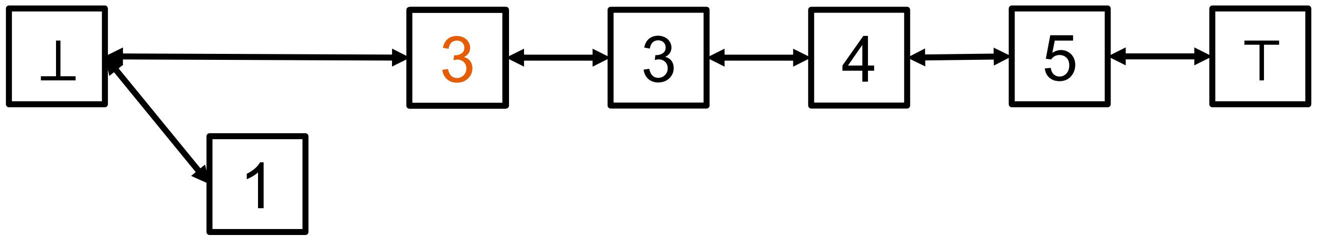

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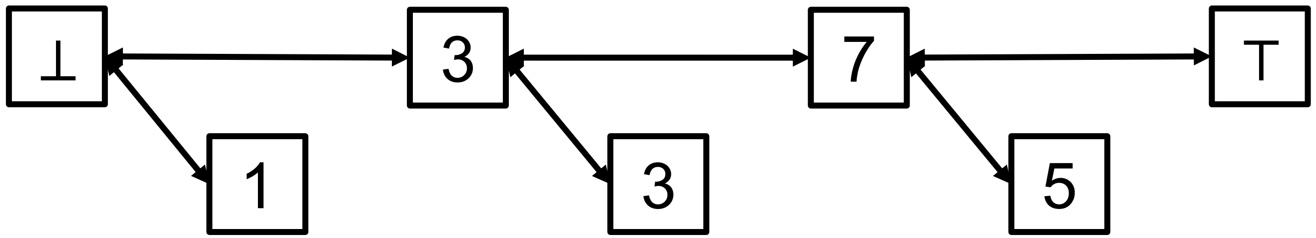

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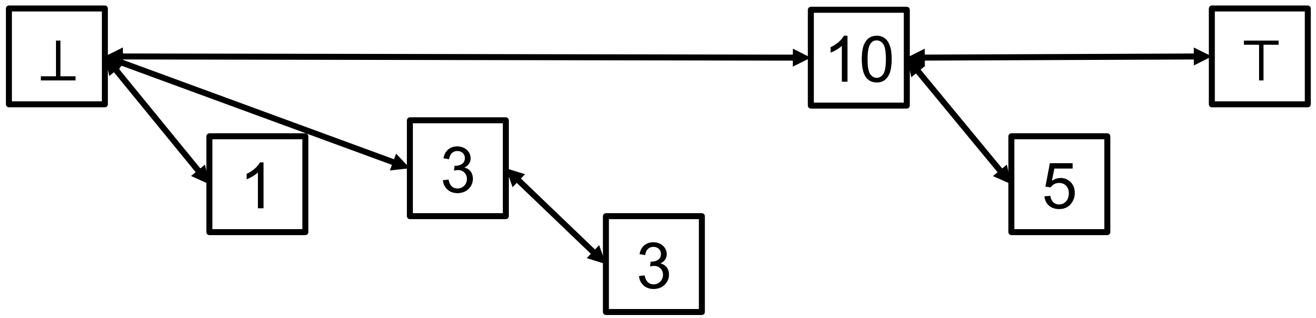

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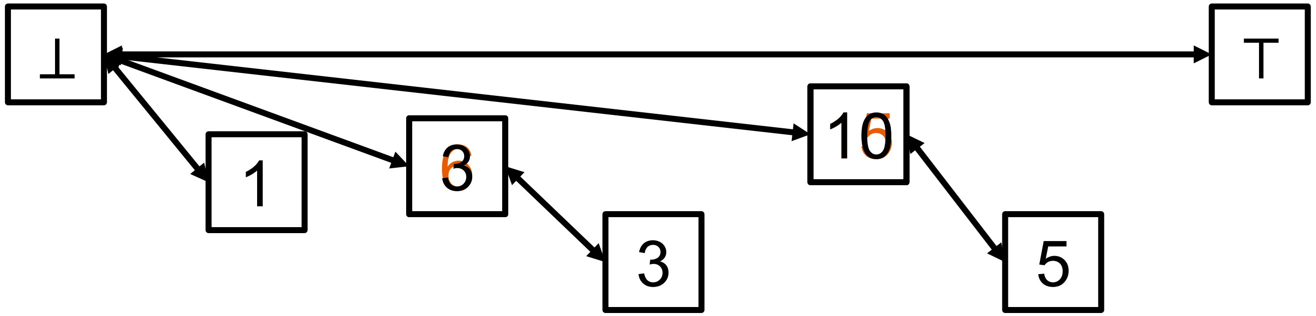


List contraction + Reconstruction = List Ranking

Use splice operation to contract the list and generate a tree with shallow (ideally $O(\log n)$) depth

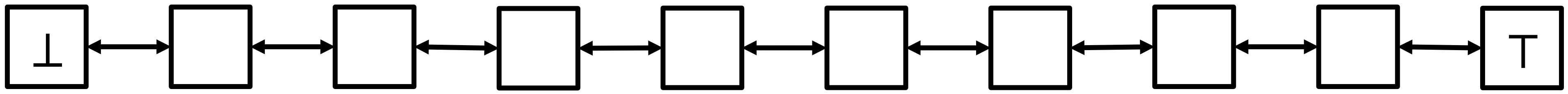
List contraction

- Have been studied for decades **[AM13, Baase93, CV86, JáJá92, JLS14, KR90, Ranade98, RMM93, SGB⁺15, Vishkin84, Vishkin93, Wyllie79]**
  - The optimal ones (on PRAM) are complicated
  - The practical ones are not theoretically efficient
  - All existing algorithms requires $\Omega(\log n)$ rounds of global synchronization

- Goal: simple, both theoretically and practically efficient algorithm, using no global synchronization

Motivation from [SGB+15]

- Conceptually simple and practically efficient



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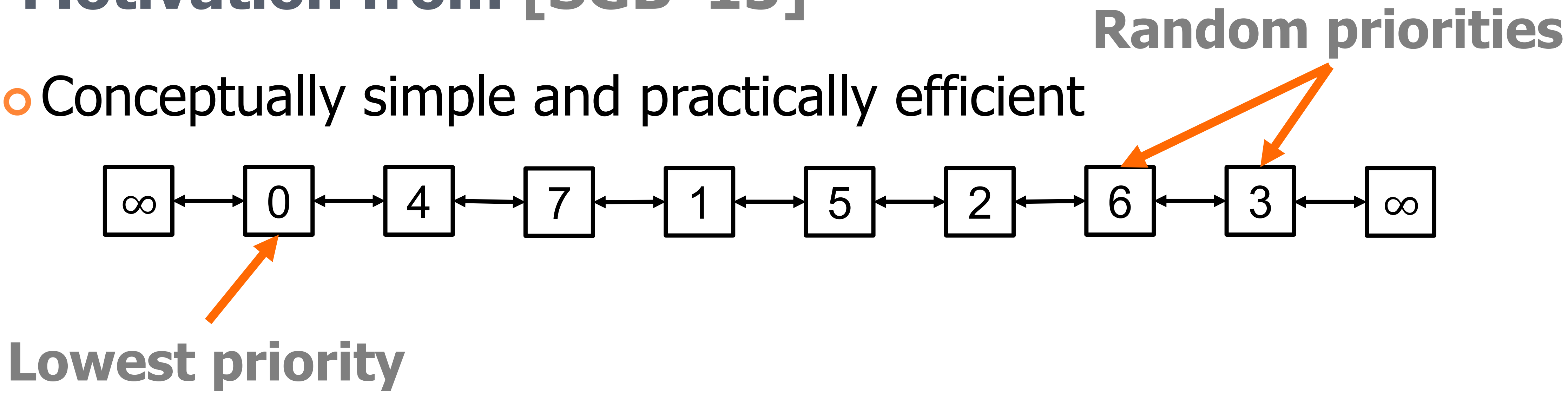

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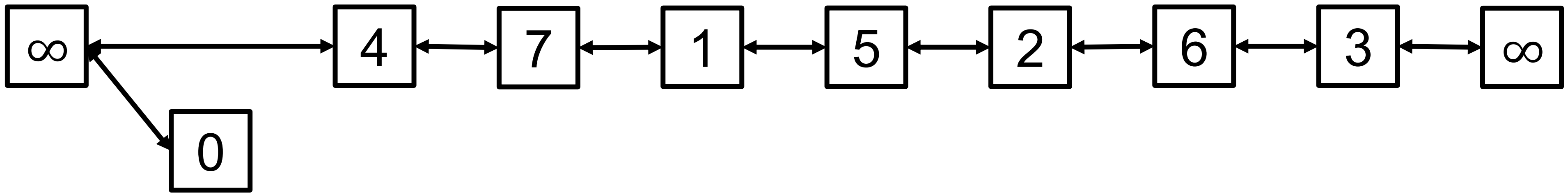

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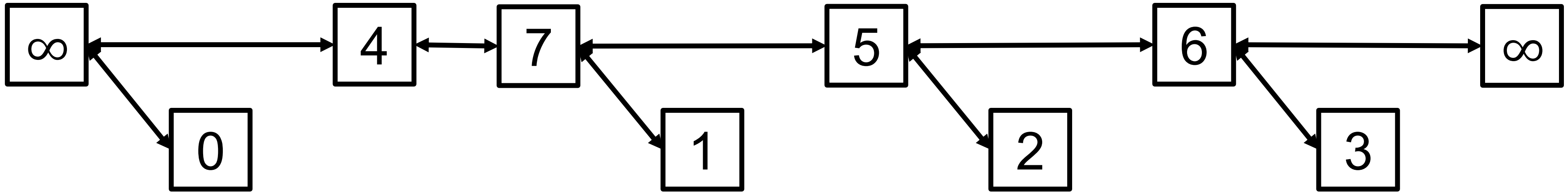
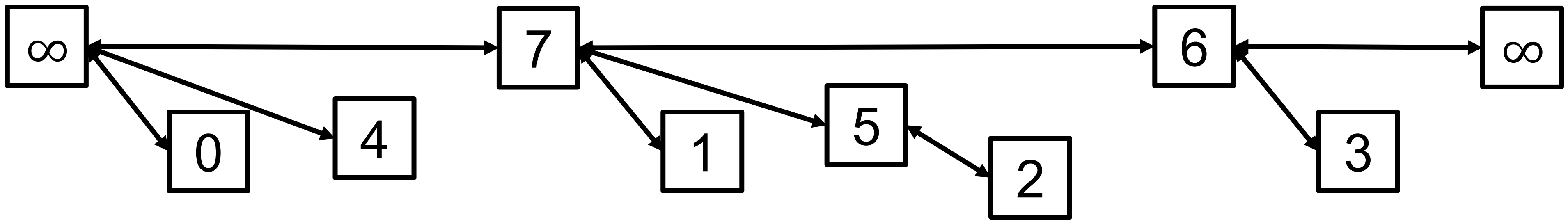


Round 1

Motivation from [SGB+15]

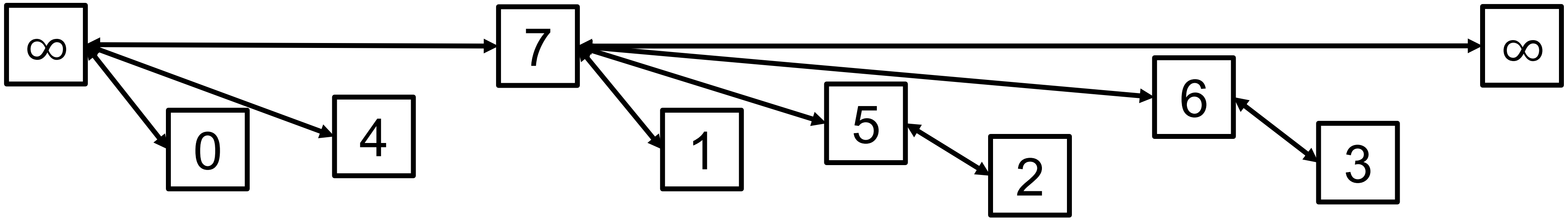
- Conceptually simple and practically efficient



Round 2

Motivation from [SGB+15]

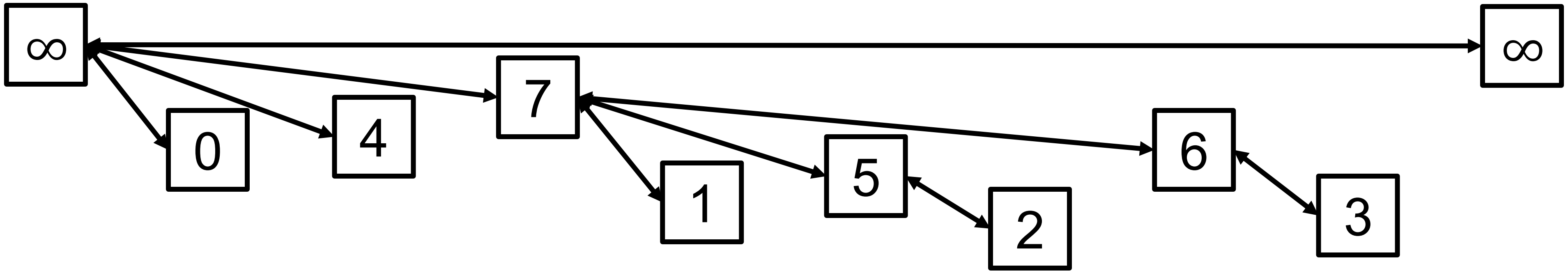
- Conceptually simple and practically efficient



Round 3

Motivation from [SGB+15]

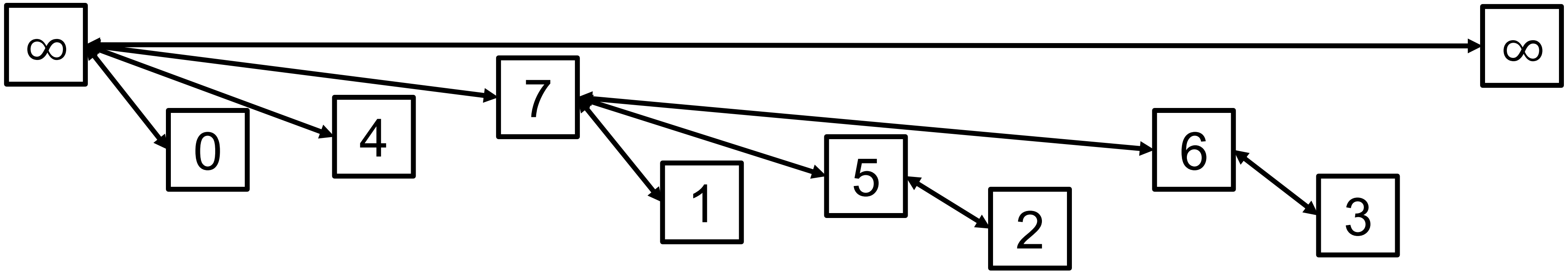
- Conceptually simple and practically efficient



Round 4

Motivation from [SGB+15]

- Conceptually simple and practically efficient



Give a random priority to each node

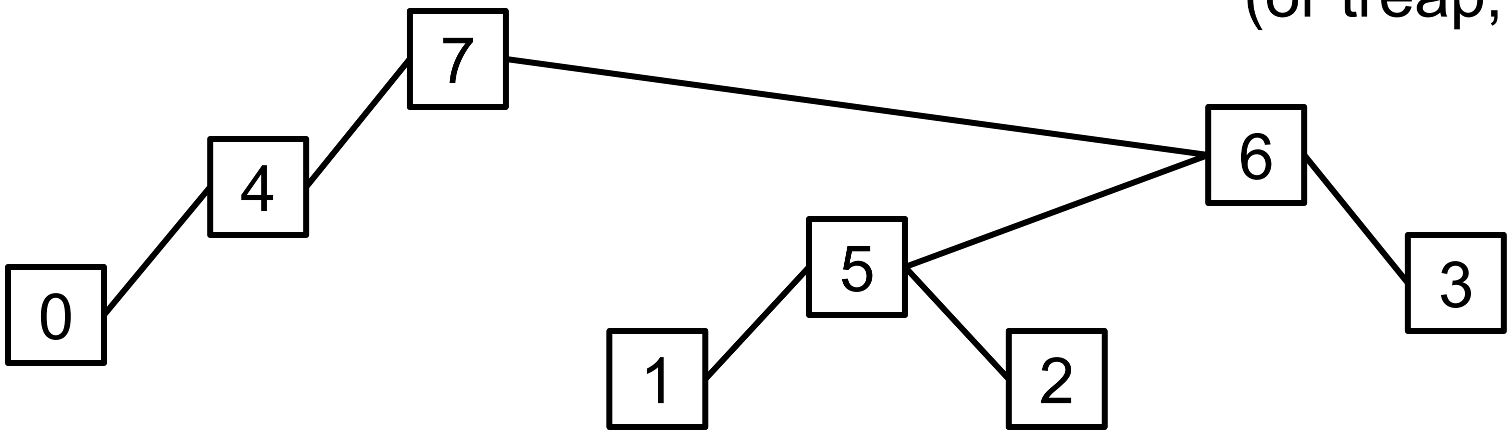
While not fully contracted

Contract all feasible nodes in parallel

Pack the rest of the nodes

Dependency between splices

Tree height is $\Theta(\log n)$ *whp* [SGB+15], the same as a randomized BST (or treap, quicksort)



Give a random priority to each node

While not fully contracted

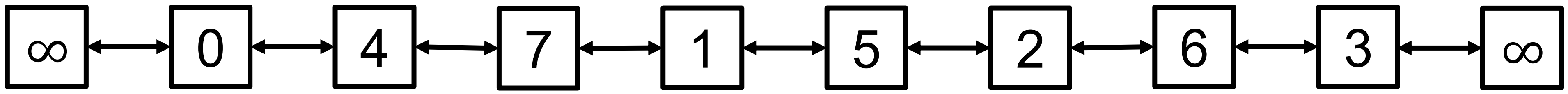
Contract all feasible nodes in parallel

Pack the rest of the nodes

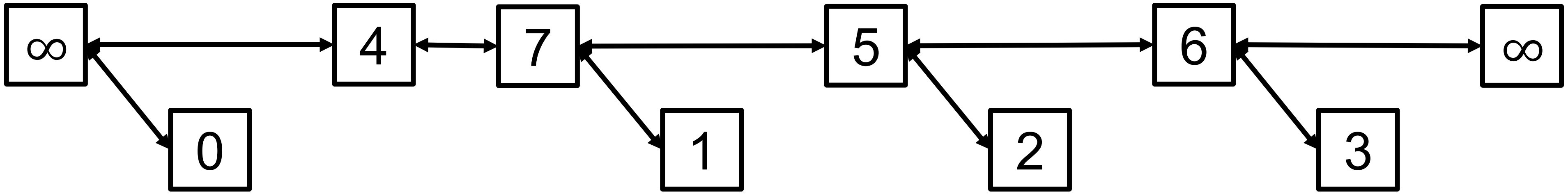
Disadvantages:

- $\Omega(\log n)$ rounds of synchronization
- Need to pack in every round

Asynchrony in splicing

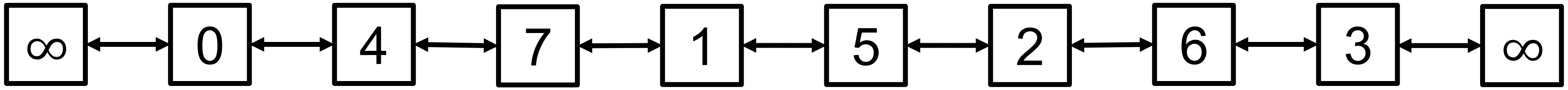


Asynchrony in splicing

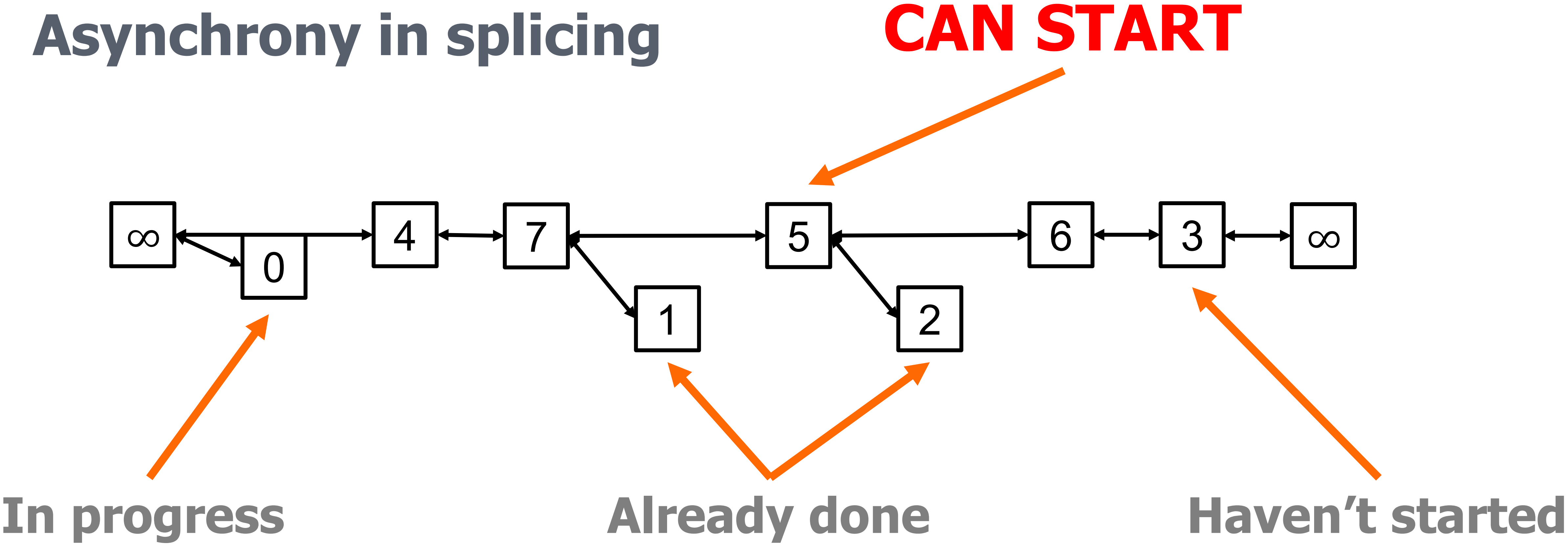


Round 1

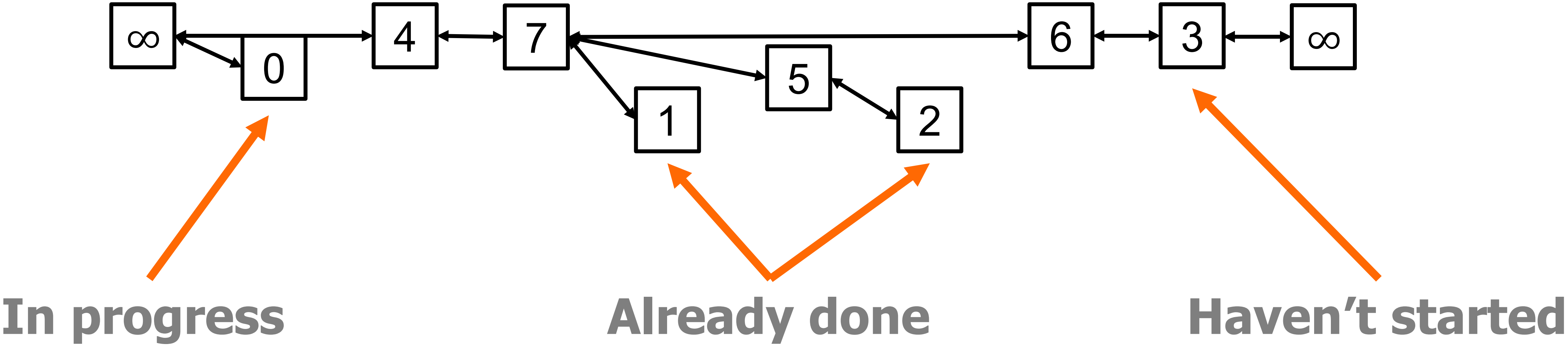
Asynchrony in splicing



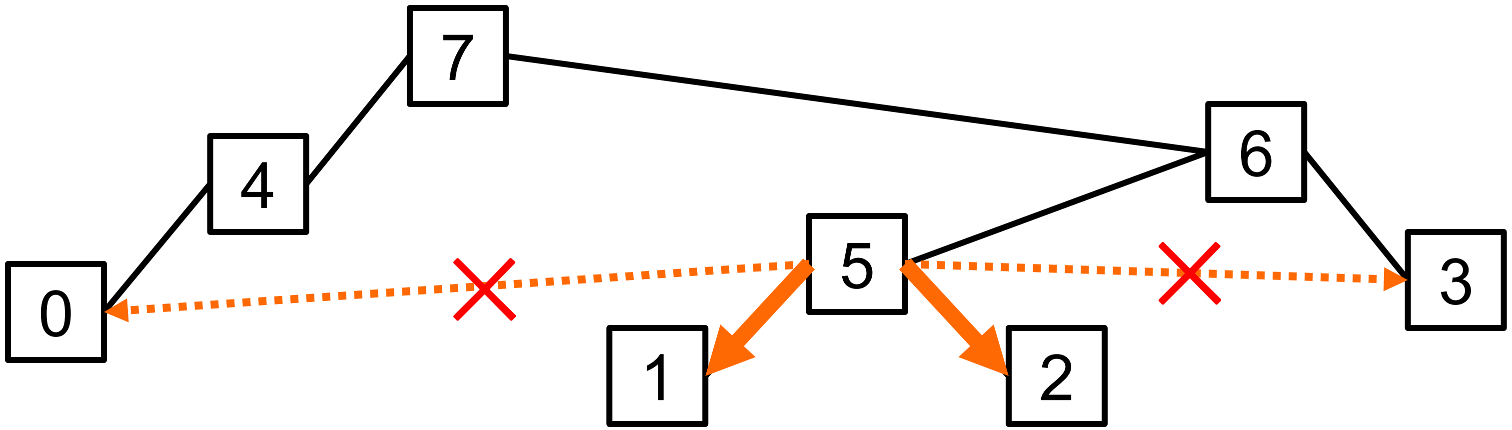
Asynchrony in splicing



Asynchrony in splicing



New list contraction algorithm



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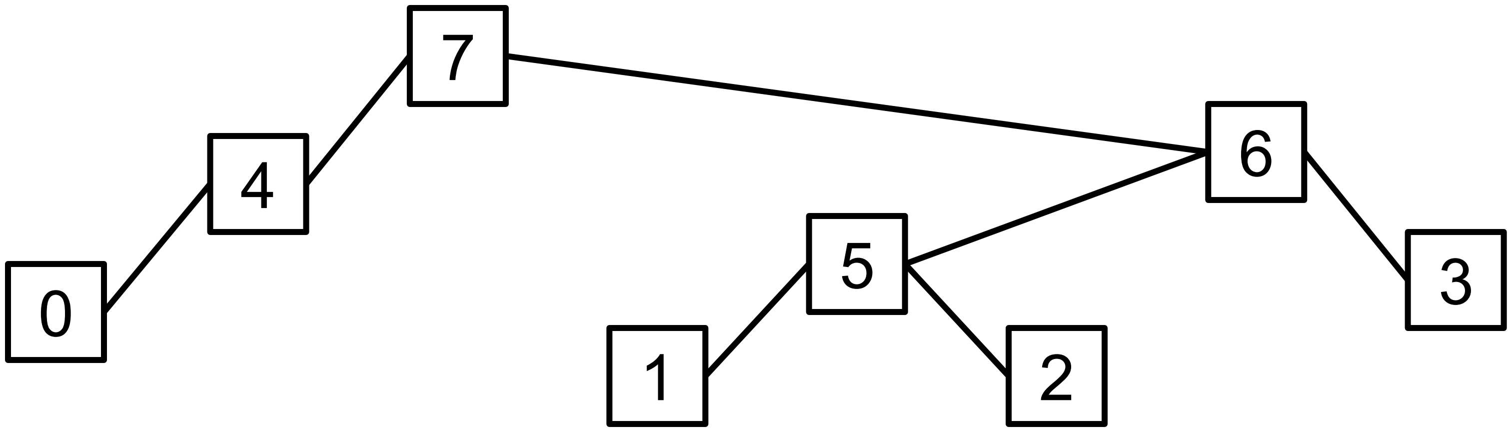
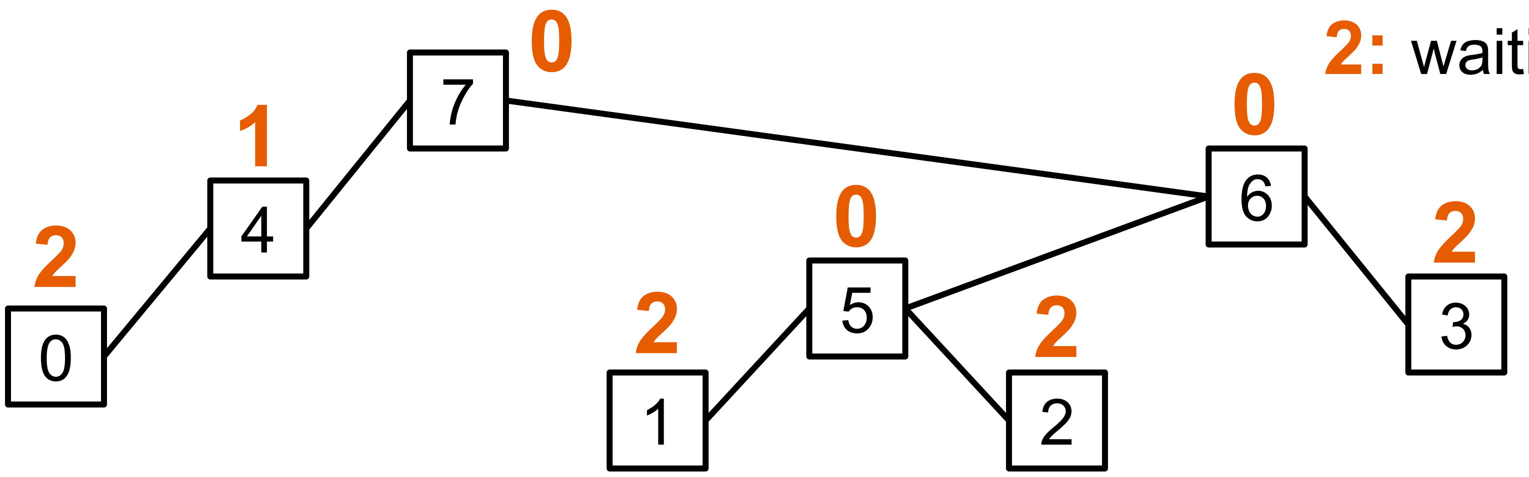


set the flag for each node
parallel-for each node v **do**
    if v is leaf **then**
        while list is not empty **do**
            splice v out
            $v \leftarrow v$.prev or v.next with smaller priority
            if `Test&Set`(v.flag) then break

New list contraction algorithm

0: waiting for two children
1: waiting for one child
2: waiting for no children



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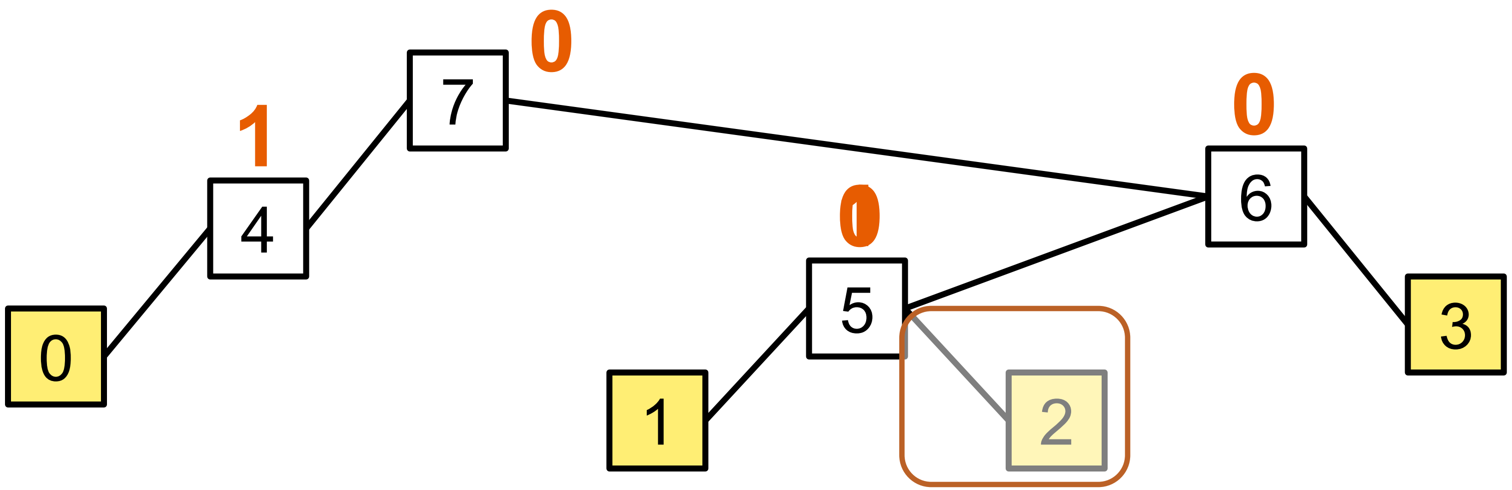

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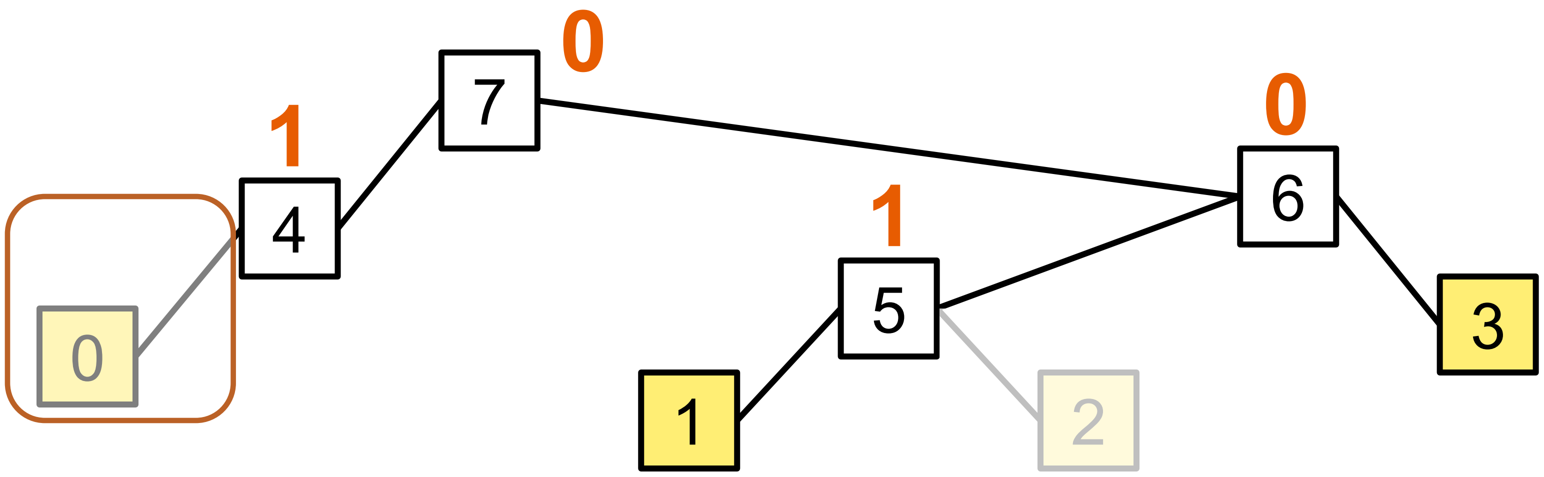

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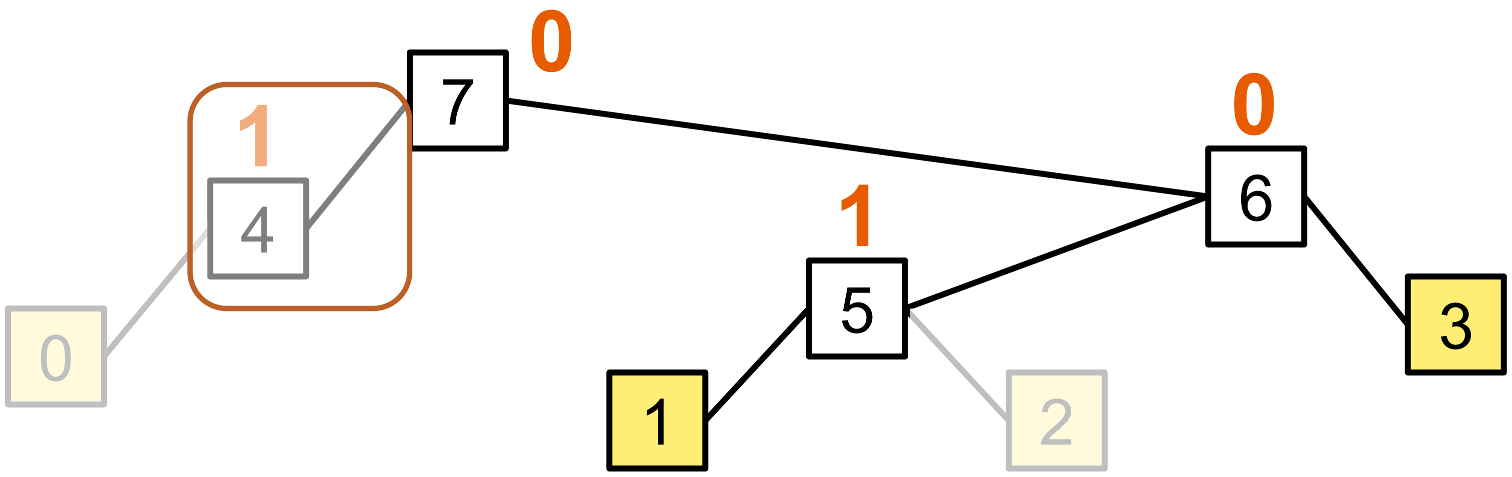

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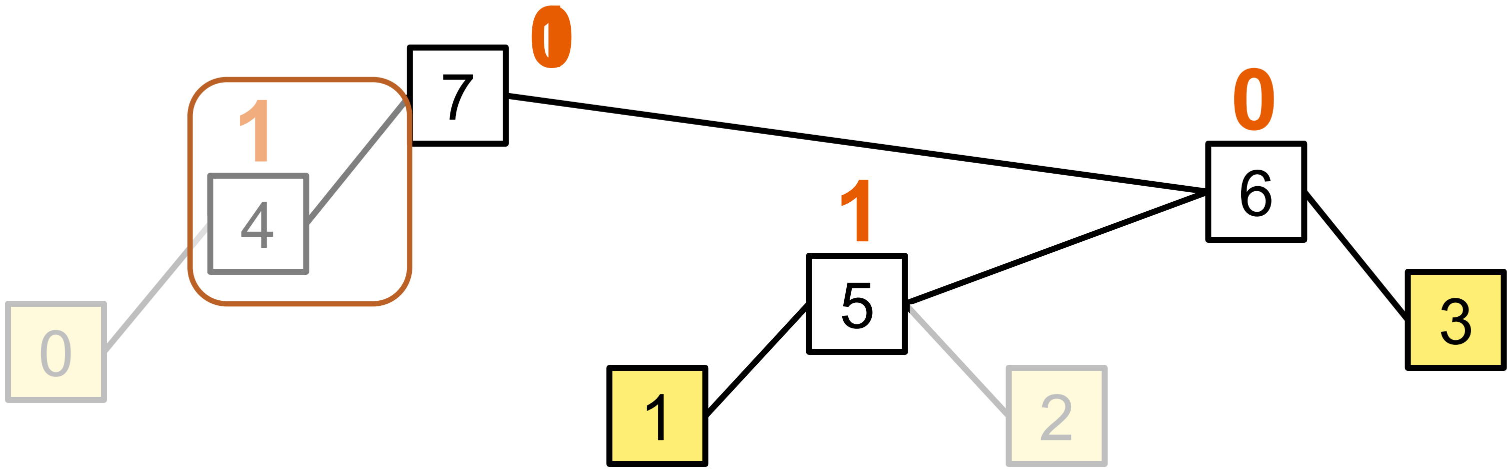

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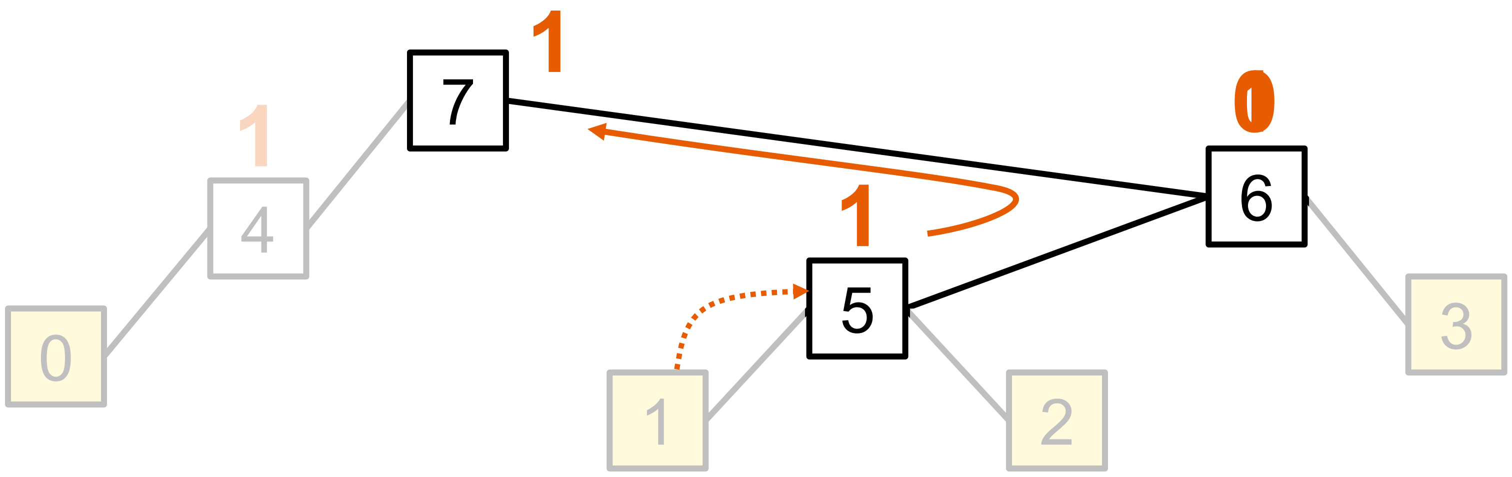

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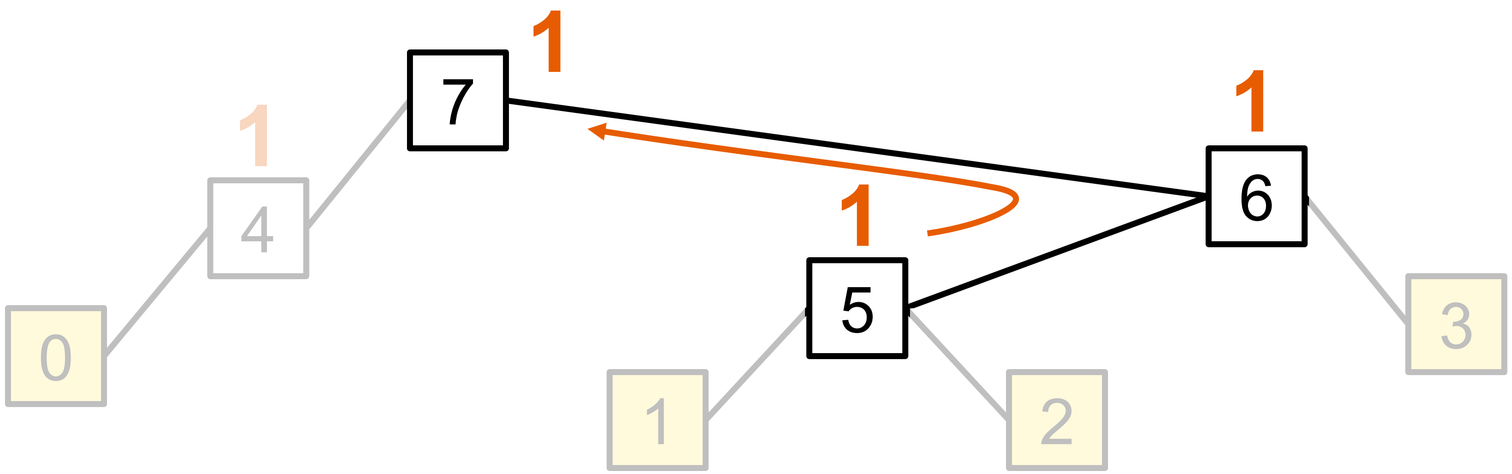
No packing

No global synchronization

New list contraction algorithm

0: waiting for two children
1: waiting for one child



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Worst-case

Work: $O(n)$

Span: $O(\log n)$ *whp*

Extremely simple to implement (<20 lines)

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```
void listRanking(node *A, intT n) {
  char* R = new char[n];
  parallel_for (intT i=0; i < n; i++)
    R[i] = (A[i].pri < A[A[i].prev].pri) || (A[i].pri < A[A[i].next].pri);
  parallel_for (intT i=0; i < n; i++) if (R[i] <= 1) {
    intT c = i, prev = A[c].prev, next = A[i].next;
    if ((A[i].pri < A[prev].pri) && (A[i].pri < A[next].pri)) {
        while (A[c].prev != n || A[c].next != n) {
            A[next].prev = prev;
            A[prev].next = next;
            R[c] = 2;
            c = A[prev].pri < A[next].pri ? prev : next;
            if (utils::CAS(&R[c], (char)0, (char)1)) break;
            prev = A[c].prev; next = A[c].next;
        }
    }
  }
  delete[] R;
}
```

More efficient than the highly optimized code in PBBS from [SGB+15]

Input size (million)	Running time (s)		Improved by
	New algorithm	PBBS version	
10	0.039	0.082	110%
20	0.094	0.16	70%
50	0.23	0.38	65%
100	0.51	0.72	41%
200	1.11	1.53	38%
500	2.85	3.85	35%

g++ with Cilk
on 72 cores

Saved by avoiding **synchronization**
(more when input is small)

Saved by avoiding **packing**
(consistent for all cases)

Summary for the new list contraction algorithm

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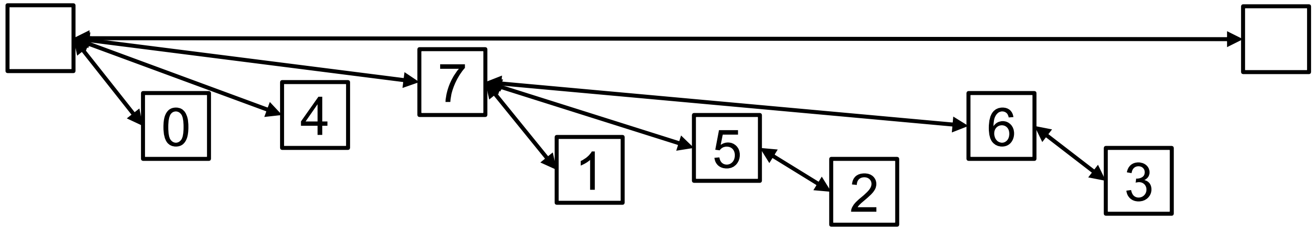
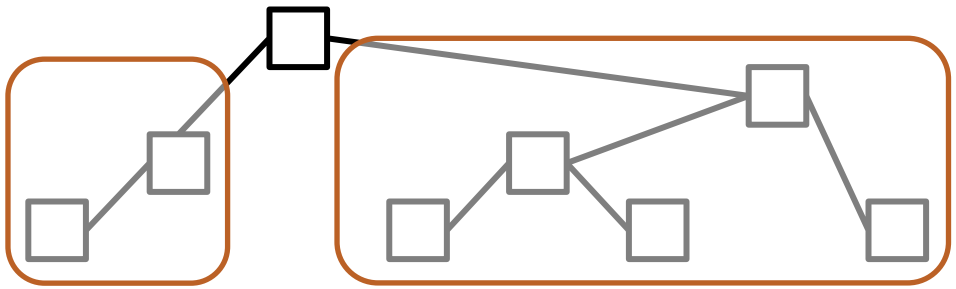


- Simple both conceptually and practically (rely on `Test&Set`)
- Efficient both theoretically and practically
- Highly asynchronous and not round based

Why so simple? Anything more general?

- Challenge in parallelism: dependencies
  - Classic solution: divide-and-conquer, round-based



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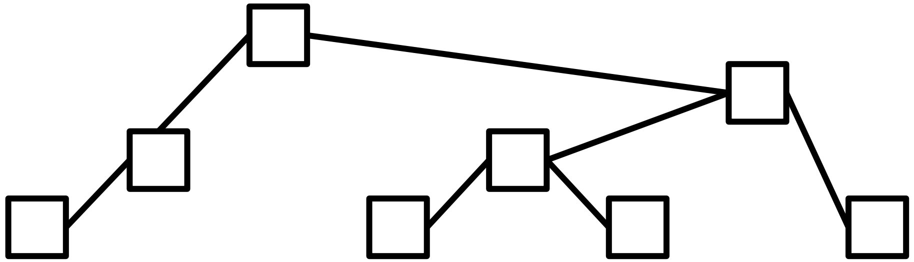
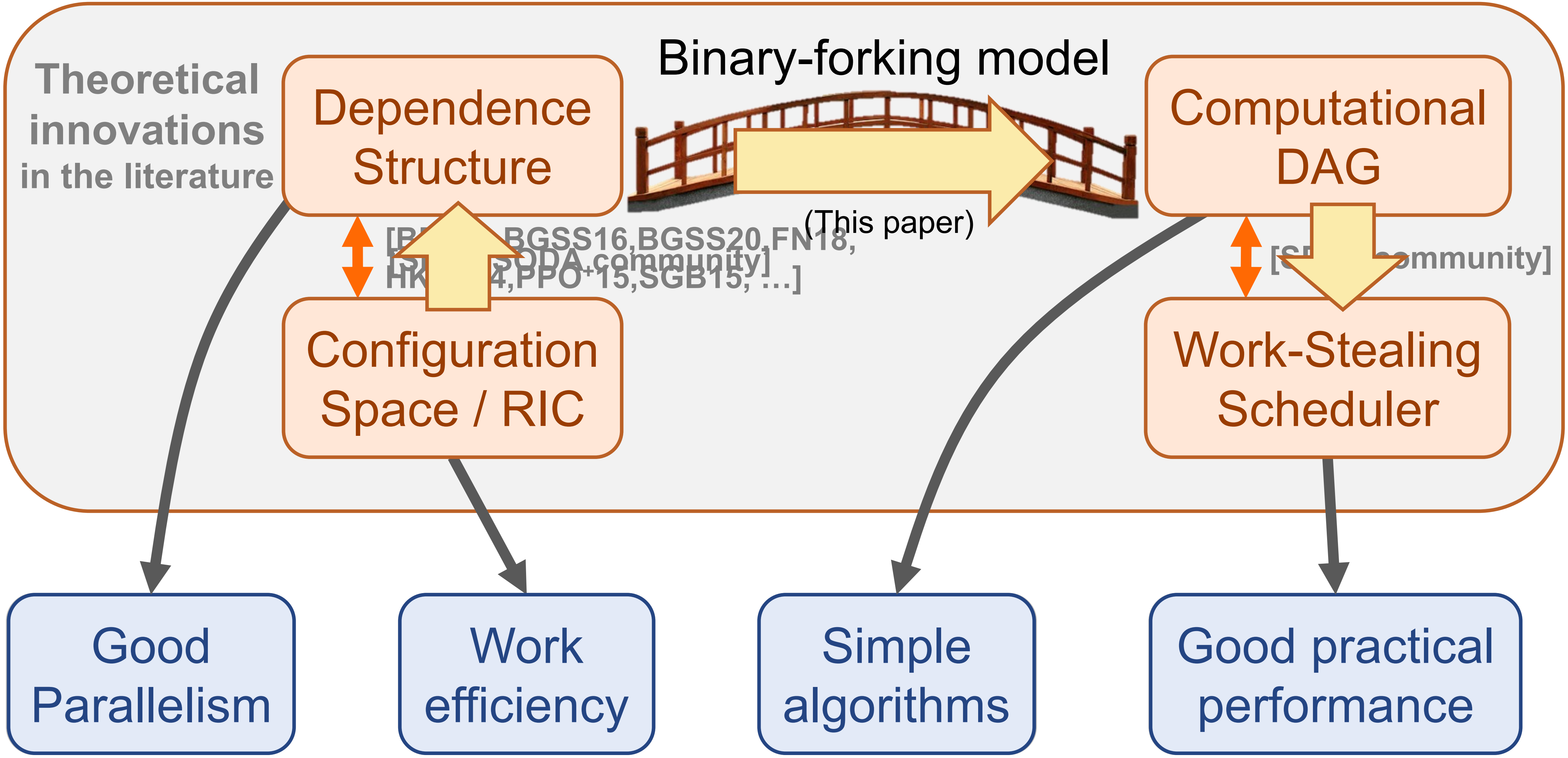


- Challenge in parallelism: dependencies
  - Classic solution: divide-and-conquer, round-based
- What if we directly convert dependence structure to the computational DAG?
  - The rest of the work is done by the work-stealing scheduler!
  - Works for constant dependencies per node via `Test&Set`
  - Simple algorithms: List contraction, tree contraction, random permutation, sequence algorithms (e.g., RMQ), dynamic programs
- Idea combined with configuration space [Mulmuley 1994] solved **open problems**: incremental Delaunay triangulation [JACM 2020], convex hull [SPAA 2020] (a general framework for analyzing this type of algorithms)

A new angle for (asynchronized) parallel algorithms



Other optimal parallel algorithms in the BF model

Problem	Work	Span/ depth
List contraction	$O(n)$	$O(\log n)^*$
Sorting	$O(n \log n)^{\uparrow}$	$O(\log n)^*$
Semisort	$O(n)^{\uparrow}$	$O(\log n)^*$
Random permutation	$O(n)^{\uparrow}$	$O(\log n)^*$
Range minimum query	$O(n)$	$O(\log n)$
Tree contraction	$O(n)$	$O(\log n)^*$
Ordered-set operations (Union, intersect, diff)	$O\left(m \log\left(\frac{n}{m}+1\right)\right)$	$O(\log n)$

$^{\uparrow}$ expected, * with high probability

Complexity results

- Let $\mathcal{BF}^1$ be the class of algorithms requiring $O(\log n)$ depth/span and polynomial work in the BF model

$$\mathcal{NC}^1 \subseteq \mathcal{L} \subseteq \mathcal{BF}^1 \subseteq \mathcal{AC}^1 \subseteq \mathcal{PRAM}^1_{CRCW} \subseteq \mathcal{NC}^2$$

- Let $\mathcal{BF}(W(n), S(n))$ be the class of algorithms requiring $S(n)$ depth/span and $W(n)$ work

$$\mathcal{BF}_{\text{Fork-join}} \subsetneq \mathcal{BF} \subseteq \mathcal{BF}_{CAS}$$

↑

[Manuscript from GJS]

Modern parallel machines are highly asynchronous, and global synchronization is costly and should be avoided if possible

1 The binary-forking model and variants

Our suggested model to design and analysis algorithms, and related complexity results

2 Optimal algorithms in the new model

All use no or constant rounds of global synchronization
Many are surprisingly simple

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Future work on exploring asynchrony in parallel algorithms

Complexity, algorithms, implementation, software, etc.

Modern parallel machines are highly asynchronous, and global

Full version of this talk (40 min):

https://www.cs.ucr.edu/~ygu/video/bf.mp4

(or just search "Yan Gu UCR")

Motivations from system and architecture (1:00 – 3:20)

Benchmarking global synchronization cost (4:30 – 6:30)

More discussion for multithreaded models (11:30 – 18:00)

Complexity theory (18:00 – 19:00)

Related work in the BF model (19:00 – 21:20)

List contraction in more details (22:00 – 33:00)

How does this idea generalize to other problems (33:00 – 36:00)

Comparison sort and ordered-set operations (36:40 – 38:00)

Various anecdotes (random places)